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International Center for Public Policy
Andrew Young School of Policy Studies
Georgia State University
Atlanta, Georgia 30303
United States of America

Phone: (404) 413-0235
Fax: (404) 651-4449
Email: paulbenson@gsu.edu
Website: http://icepp.gsu.edu/

International Center for Public Policy
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Asymmetric Financial Indexation and Speculative Rational Price Bubbles in the Housing Market

Mario I. Valenzuela-Silva,[1] Jorge Martinez-Vazquez,[2] and José L. Sáez-Lozano[3]

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Abstract

This paper analyzes the existence and persistence of residential price bubbles where housing and financial markets have asymmetric price indexation mechanisms, with an application to the case of Chile. Such a design of market institutions presents a unique dichotomy: while real estate asset prices and mortgages are usually expressed in real terms (i.e., indexed to the inflation rate), close substitutes such as house rents are expressed in nominal terms or under imperfect indexation mechanisms. This asymmetry can induce an additional risk of investing in residential assets, which in turn would put upward pressure on risk premiums in real estate investments, through higher requirements and expectations of overpricing and capital appreciation. In this paper we hypothesize that such asymmetry in the price trajectory, in a highly integrated market, favors the formation and persistence of rational speculative price bubbles. In the face of changing expectations or possible elimination of financial indexation, residential bubbles may fade away, triggering high social costs. Our empirical work uses Bayesian analysis to estimate a general equilibrium SVARX model for the new housing market using data for Chile between January 2003 and June 2022. The results validate the proposed hypothesis on the formation and persistence of rational speculative price bubbles in the presence of asymmetric price indexation mechanisms. The policy implication is that partial financial indexation, not extended to the rest of the economy, is not always desirable given the risk of significant price imbalances it may cause, especially in highly integrated markets such as finance and real estate.

Keywords: housing bubbles, market dichotomy, asymmetric indexation, Bayes estimation, over-indebtedness, over-expectations

JEL Classification: D14, E44, G21, R21

[1] Faculty of Economic and Business Sciences, Office B-301, Campus Universitario de la Cartuja 18071, Granada, Spain. Phone: 958420927. e-mail: mvalenzuelas@correo.ugr.es

[2] Faculty of International Center for Public Policy, Georgia State University, Atlanta, USA. Phone: 7706537960. e-mail: jorgemartinez@gsu.edu

[3] Faculty of Economic and Business Sciences, office B-301, Campus Universitario de La Cartuja 18071 Granada, Spain. Phone: 958240927. e-mail: josaez@ugr.es

1. Introduction

Little is known and written about the relationship between price indexation mechanisms and asset price bubbles. Indexation mechanisms are used with some frequency—for example, to keep real wages or real exchange rates stable, or even to adjust tax burdens and charges for basic services. Less frequent is the asymmetric indexation of prices in asset markets such as housing and rents, which are closely linked to the mortgage credit market, sometimes indexed to the inflation rate. As we shall see, these indexation policies and practices have the potential to influence the development of rational speculative price bubbles.

The objective of this paper is to fill the gap in the economic literature on the relationship between price indexation mechanisms and asset price bubbles in real estate markets. The relevance of this study from a policy perspective is to avoid the social cost and wealth transfers associated with an abrupt adjustment in price imbalances because of the elimination or changes in financial indexation mechanisms, or changes in the rational expectations of economic agents about such indexation. From a more general monetary policy perspective, this paper raises the questions of what the optimal levels of financial indexation are and to what extent the economic authority should contribute to avoid the formation of asset price bubbles through the better design of the indexation mechanisms themselves.

There are two fundamental issues to be addressed in economic and financial indexation. First, what is the impact on the relative price structure and, therefore, on the optimal allocation of resources, particularly when the levels and speed of indexation are uneven or asymmetric? Second, what is the effect on the behavior and risk aversion of economic agents in highly integrated markets especially when, as is commonly the case, the single indexation mechanism for prices is partial rather than total? It may be idealistic to seek total indexation, however, both because of its practical impossibility and because of the loss of effectiveness that monetary policy could suffer in controlling inflation.

In Chile, financial indexation has historically been important for, among other things, encouraging private savings, and from 1967 to 2001, the main objective of monetary policy was to influence liquidity based on the real interbank interest rate (i.e., corrected for the inflation rate). Since 2001, although monetary policy has used a nominal interbank rate, it has not been possible to discontinue the practice of indexing the value of mortgage loans and the prices of new and old houses. By contrast, the value of rents has remained expressed in nominal terms or with only discrete and partial adjustments. Specifically, in the capital city of Santiago, between January 2003 and June 2022, the real price of new housing more than doubled (131.0 percent), a much higher rate of growth than that in the real value of house rents (8.2 percent). Assuming housing is an investment, the opportunity cost of investing in them, as determined by the real price of the most traded shares in the capital market, grew by only 63 percent. This is the basic data we use to determine the relationship between partial financial indexation, embodied in an asymmetric path between housing prices and real rents, and the formation of rational speculative price bubbles.

The remainder of the paper is organized as follows. The next section reviews the existing literature on price indexation and asset price bubbles. In section 3 we develop a theoretical framework for how the nature of the price indexation process in the housing market can lead to price bubbles. Section 4 postulates the working hypothesis, empirical model, estimation methods, and data. Section 5 presents results, including stability and parametric convergence tests. The last section concludes and suggests future lines of research.

2. Literature review

The issue of economic and financial indexation is particularly relevant if we consider the proposal of Robert J. Shiller (2002) who, after visiting Chile and following Shiller, Schultze, and Hall (1997), postulated that indexation can be extended to all prices in an economy, including that of the United States, despite the relatively low inflation rates in that country. According to Shiller (2005), this would improve the dynamics of the relative price structure while discouraging any loss of inflationary control. In our opinion, this important, bold proposal has implications and hidden potential costs that have not yet received due consideration in the economic literature.

The scarce existing literature on indexation and asset price bubbles comes from a series of recent works on the oil and natural gas market, whose destination is mainly Europe, North America, and Asia [e.g., Grandi (2014), Komlev (2016), Shi and Variam (2016), Su, Li, Chang, and Lobonţ (2017) and Zhang, Wang, Shi, and Liu (2018)]. These are markets with oligopolistic structures and characteristics, such as cartels and implicit or explicit price agreements that influence the formation of rational speculative price bubbles, with prices beyond what cost and relative scarcity fundamentals would justify. Such imbalances would be influenced by speculative and discretionary factors of a few bidders, rather than by generalized speculative demand behavior as described by Shiller (2014).

A study that addresses the possible relationship between financial indexation and speculative residential price bubbles is Miles and Pillonca (2008). These authors analyze speculative demand behavior for housing prices in Europe between 1996 and 2006. This study identifies mortgages indexed to the inflation rate, and not to house prices, in countries such as Spain, Sweden, Belgium, and the United Kingdom. The analysis highlights the speculative role played by homebuyers' expectations of future capital appreciation gains, which could lead to rational house price bubbles beyond what fundamentals such as real income and population would indicate. However, this pioneering work stopped short of quantifying the magnitude of a potential price bubble, nor did it delve into the risks associated with asymmetric price indexation systems.

A second paper suggesting a possible relationship between indexation and real estate price premiums is Campbell and Cocco (2003). Based on a life-cycle model of optimal mortgage choice, under uncertain inflation due to high price volatility, these authors show that inflationary indexation of prices and mortgages and the establishment of a fixed real interest rate on mortgages would be attractive. But, while the use of real values would provide inflationary insurance on equity for households and investors, it could also encourage speculative behavior on the expected future trajectory of real house prices. However, this study also stops short of addressing the presence and magnitude of housing price bubbles.

On a more distant but related topic, there is also a wide variety of opinions on the role that central banks should play in preventing real estate bubbles. Although Taylor (2007) acknowledges that monetary policy in the United States has helped to moderate the rise in housing prices, the low interest rates applied between 2002 and 2005 would have encouraged an over-demand for housing, which was reinforced by bad credit risk practices. Gordon (2009) agrees that prolonged low interest rates influenced the formation of the housing bubble leading to the subprime crisis. In contrast, Dokko et al. (2009) argue, based on a structural macroeconomic model for U.S. prices, that real estate development would not have been much different following the *Taylor rule* (Taylor (1993)).[1]

After the subprime crisis, Bernanke (2010) insisted that the origin of real estate price bubbles would be mainly related to regulatory problems in the mortgage credit market. Blot, Hubert, and Labondance (2017) are more skeptical on this linkage, concluding that there is an asymmetry in the way monetary policy acts on asset prices. While expansionary monetary shocks promote price hikes and bubbles, contractionary monetary shocks would not be able to reduce or burst these imbalances. Finally, Bernanke, Laubach, Mishkin, and Posen (2018) conclude, after exploring various scenarios and international experiences, on the inability of monetary policy to monitor asset prices and prevent bubbles. In the face of the above, there are papers that consider it fundamental for a central bank to consider both price stability and financial stability as a policy objective (Brunnermeier and Oehmke (2013) and Amador-Torres, Gomez-Gonzalez, and Sanin-Restrepo (2018)).

From the above discussion, it is clear that there is no consensus in the literature as to where the main responsibility for the generation of the *subprime crisis* lies—hypotheses range from lack of banking regulations to unwise monetary policy management. They have in common the suggestion of economic policy failures, which may also be critical for the role of real estate price indexation in residential bubbles.

[1] The rule establishes the necessary level of interest rates (r) to achieve a desired equilibrium between the inflation rate (π) and the economic growth rate ($G\dot{D}P$). Assuming a neutral interest rate (r^*) that generates neither inflation nor unemployment, both policy objectives could have equal priority (0.5). Thus, if expected economic growth is given by ($G\dot{D}P^e$), long-term trend or potential growth is ($G\dot{D}P^p$), the expected inflation rate is π^e and the target inflation rate is π^o, the rule posits to look for a neutral interest rate (r^*) such that r = r^* + [0. 5($G\dot{D}P^e$ - $G\dot{D}P^p$) + 0.5(π^e - π^o)]. In case of above expected economic growth (($G\dot{D}P^e$) > ($G\dot{D}P^p$), possibly at the cost of a higher level of inflation (π^e > π^o)], the rule suggests increasing the policy interest rate (r^*) so as to moderate aggregate spending, the inflation rate and economic growth, and vice versa.

3. A simple framework for how financial indexation can lead to bubbles: An application to Chile

Financial indexation in Chile comes from the use of the "*Unit of Foment*" (UF), created in 1967. The UF is a unit of account expressed in Chilean peso units (CL$) that is adjusted daily according to the inflation of the Consumer Price Index (CPI) of the previous month, constructed by the National Statistics Institute (NSI). The UF has been used in asset and liability contracts in the financial system. Its original purpose was to encourage long-term private savings, free from inflationary erosion, to finance the placement of mortgage loans.

Thus, the UF is calculated as follows,

$$UF_d = UF_{d-1} \ (1 + \pi_{t-1})^{1/dd} \tag{1}$$

Where UF_d is the value of the unit of foment on day *d* of the current month *t,* π_{t-1} is the inflation of the previous month *t-1,* and *dd* represents the number of days of the previous month.

Considering the ex-post version of the Fisher (1930) equation, the long-term real interest rate of the month (t) of mortgage loans (rr_t) is represented by the nominal interest rate (i_t) deflated by the observed inflation rate (π_t):

$$(1 + rr_t) = (1 + i_t)/(1 + \pi_t) \tag{2}$$

Since equation (1) is invariant through time, we can represent equation (3) as follows:

$$(1 + rr_t) = (1 + i_t) \ / \ \left[\frac{UF_d}{UF_{d-dd}}\right] \tag{3}$$

From equation (3) we can deduce the effect of indexing the real mortgage interest rate on the value of mortgage loans $({M_t}^{UF})$ and on mortgage dividends $({Div_t}^{UF})$, because by simple financial mathematics:

$$Div_t{}^{UF} = M_t{}^{UF} \frac{(1 + rr_t)^n \, rr_t}{((1 + rr_t)^n - 1)} \tag{4}$$

where the total amount of the mortgage loan is only a fraction λ (0 < λ<1) of the real housing price (Pv_t^{UF}) that is financed by a bank in UFs:

$$M_t^{UF} = \lambda \, Pv_t^{UF} \tag{5}$$

This demonstrates the way in which financial indexation using the UF has permeated the real estate market through the mortgage credit market. Real estate developers publish the prices of new homes in UF, a practice extended to the old housing market, forcing homebuyers to express their preferences and willingness to pay in this unit.

However, the value of monthly rents, as well as most of the rest of the prices, continues to be traded in nominal terms with real decreases in the presence of inflation (CL$). Although there are leases that establish contractual clauses for semi-annual or annual adjustments for inflation, in practice they are rarely used due to, among other things, the risk of loss or non-payment by tenants. Such mismatch with real housing prices generates a growing gap over time, exposing a systematic aversion and investment risk given by nominal rent flows. This is manifested in higher-risk premiums demanded by investors in the sale of their assets, who are backed by UF mortgages, and the push for higher sale or resale prices to compensate for their risk aversion.

Considering housing as a financial asset that has a real price and a stream of nominal rental income (Campbell and Shiller (1988)), we can analyze the mismatch by considering the rate of return (r_{t+1}):

$$r_{t+1} = \left[\frac{Pv_{t+1}^{UF} - Pv_t^{UF} + R_t^{CL\$}}{Pv_t^{UF}} \right] \tag{6}$$

Clearing for price:

$$Pv_t^{UF} = \left[\frac{Pv_{t+1}^{UF} + R_t^{CL\$}}{(1 + r_{t+1})}\right] \tag{7}$$

Taking the expected value of equation (7), considering all the information available at *t*:

$$Pv_t^{UF} = E_t\left[\frac{Pv_{t+1}^{UF} + R_t^{CL\$}}{(1 + r_{t+1})}\right] \tag{8}$$

And reproducing equation (8) for *k* periods onwards, we obtain:

$$Pv_t^{UF,CL\$} = E_t\left[\sum_{i=1}^{k}\left(\frac{1}{(1 + r_{t+i})}\right)^{i} R_{t+i}^{CL\$}\right] + E_t\left[\left(\frac{1}{(1 + r_{t+k})}\right)^{k} Pv_{t+k}^{UF}\right] \tag{9}$$

It can be seen from (9) that the future price of housing (Pv_{t+k}^{UF}) as a residual value, should tend to zero in efficient markets, since in the long run prices should be determined only by the expected rental flows:

$$\lim_{k\to\infty} E_t\left[\left(\frac{1}{(1 + r_{t+k})}\right)^{k} Pv_{t+k}^{UF}\right] \to 0 \tag{10}$$

However, the future price can hardly converge to zero when k →∞ because it is indexed to an increasing value over time such as the UF. In contrast, the present value of the future flow of nominal rents expressed in Chilean pesos usually grows below the inflation rate, which is summarized in:

$$\lim_{k\to\infty}[Pv_t^{UF}] > \lim_{k\to\infty} E_t\left[\sum_{i=1}^{k}\left(\frac{1}{(1 + r_{t+i})}\right)^{i} R_{t+i}^{CL\$}\right] \tag{11}$$

because of:

$$\lim_{k\to\infty} E_t\left[\left(\frac{1}{(1 + r_{t+k})}\right)^{k} Pv_{t+k}^{UF}\right] > 0 \tag{12}$$

Thus, the asymmetric indexation between Pv_t^{UF} and $R_t^{CL\$}$ breaks the basic principle suggested by economic theory that the price of any asset is nothing more than the present value of future income flows:

$$Pv_t^{UF} \neq E_t\left[\sum_{i=1}^{k}\left(\frac{1}{(1+r_{t+i})}\right)^{i} R_{t+i}^{CL\$}\right] \tag{13}$$

By simplification, assuming perpetuity of the constant value of real rents in UF, where $R_{t+i}^{UF} = R_{t+i}^{CL\$}/IPC_{t+i}$, it is possible to attribute this divergence or gap between both real values in each period to the *incubation of a bubble* in *t*:

$$\hat{b}_t^{UF} = \frac{\partial\left(Pv_t^{UF}\right)}{\partial t} - \frac{\partial\left(R_t^{UF}\right)}{\partial t} \lesseqgtr 0 \tag{14}$$

where the proxy to the accumulated bubble at present value $\left(\hat{B}_t^{UF}\right)$, as a component of the real observed price, would be:

$$\hat{B}_t^{UF} = E_t\left[\sum_{i=1}^{k}\left(\frac{\hat{b}_{t+i}^{UF}}{(1+r_{t+i})}\right)^{i}\right] > 0 \tag{15}$$

Thus, equation (15) can be interpreted as a proxy to the rational speculative price bubble $\left(\hat{B}_t^{UF}\right)$, since it is generated from a partial and not total indexation system of the financial and real estate sector, which encourages higher capital appreciation demands from investors.[2] As McQueen and Thorley (1994) point out, rational speculative bubbles allow prices to deviate from their fundamentals without the presence of irrational behavior. Investors internalize that prices may move away from their fundamentals but believe that there is a high probability that the bubble

[2] In the hypothetical situation where both rents and prices are equally indexed to the inflation rate, we would be faced with the classic version of equation (9), where financial theory predicts, for any type of asset, that the residual value of the price should be zero in the long term, without any opening of gaps with the present value of lease flows.

will continue to grow (under the unbalancing structural conditions already described) and that it will allow high profits, which more than offset the likely loss in the event of the bubble bursting.

Conceptually, the residential price bubble can be expressed as the difference between the price observed in the market and the price determined by its fundamentals or rental flows (Pv_t^{*UF}):

$$B_t^{UF} = \quad Pv_t^{UF} \quad - \quad Pv_t^{*UF} \tag{16}$$

where,

$$Pv_t^{UF} \quad = \quad \frac{E_t[Pv_{t+1}^{UF} \ + \ R_{t+1}^{UF}]}{(1 + \ r_{t+1}^{*UF})} \tag{17}$$

$$Pv_t^{*UF} \quad \equiv \quad \sum_{i=1}^{\infty} \frac{E_t\left[R_{t+i}^{UF}\right]}{\coprod_{j=1}^{i}\left(1 + \ r_{t+j}^{*UF}\right)} \tag{18}$$

According to Caamerer (1987), in equilibrium rational speculative price bubbles should grow on average at the same rate of return demanded by investors (r_t^{*UF}), which in this case considers a risk premium due to the asymmetric indexation of rent and dividend flows:

$$E_t[B_{t+1}^{UF}] \ = \ B_t^{UF} \ (1 + \quad r_{t+1}^{*UF}) \tag{19}$$

4. Empirical methodology

To test the hypothesis relating asymmetric indexation and bubbles, we formulate a general equilibrium model relating financial and housing markets, based on Poterba (1984), Topel and Rosen (1988), and Mankiw (2008). This is translated into a dynamic simultaneous equations model (VAR) with exogenous variables (VARX), both in its *structural* (SVARX), *reduced, and recursive* forms. We identify long-run unsustainable equilibrium situations through the housing price bubble, which is considered as an endogenous variable. Given the simultaneity, the intrinsic nonlinearity, and the absence of stationarity of most of the series, which make it difficult to identify and estimate the VAR using classical frequentist statistics, we opt for a

Bayesian cointegration estimation approach, with emphasis on the convergence of the distribution of the parameters.

4.1. General equilibrium model

The market for housing services defines the real value of rents. The demand for housing services (D_s), represented by the willingness to pay per unit($UF/M2$), is determined by the real value of rents (R) that reflects the marginal valuation of either renting or self-renting a property, the real interest rate of mortgage loans (rr), the opportunity cost given by the alternative return on financial instruments(rf), and the level of real disposable income (y_{dp}). In addition, the demand for housing services (D_s) is determined by a vector (x') that contains economic variables that can reach unbalancing values, such as the indexation gap between prices and real rents (Gap), the level of mortgage indebtedness in relation to household disposable income ($D_{H}\_Y_{dp}$), and the presumably rational expectations of economic agents regarding the future (E_x). This vector (x') may also include demographic variables such as the number of households (n_h), the average household size (m_h), and the growth rate of the labor force (p_a); the latter may be influenced by labor or humanitarian immigrations that may lead in turn to unbalancing situations. Thus, we can express the demand for housing services by:

$$D_s = D_s\left(R, rr, r_f, y_{dp}, x'\right) \tag{20}$$

The supply flow of housing services (O_s) depends on the existing housing stock (S_v), which, following Mankiw (2008), is completely inelastic and fixed in the short run ($\bar{S}_v$).

$$O_s = O_s(\bar{S}_v,) \tag{21}$$

Between equations (20) and (21), the market equilibrium real rental value for housing services (R_e) is determined, given the existing housing stock and the model's explanatory variables, conditional on the values of other economic and demographic variables:

$$R_e = R(\bar{S}_v, rr, r_f, y_{dp}, x^{'}) \quad (22)$$

Considering housing as an investment asset, the equilibrium condition indicates equality between the valuation and the marginal cost of residential capital, representing an equilibrium between the real marginal rental value generated and the *real unit use cost* (ϖ) of the existing residential capital stock for its price:

$$R(S_v, rr, r_f, y_{dp}, x^{'}) = \varpi P_v \quad (23)$$

According to Poterba (1984), the real unit use cost (ϖ) of renting a property depends positively on the real interest rate net of income tax (t), or the opportunity cost in other financial assets (rf (1- t)), the property tax (τ), and the depreciation rate (δ) that includes maintenance and wear and tear expenses, and negatively from rational expectations of appreciation per unit of capital invested in housing ($E(\dot{P}_v/z')/P_v$). Expectations are formed based on the set of information available to the investor (z') about the rest of the variables in the model:

$$\varpi = rf\ (1 - t) + \tau + \delta - E(\dot{P}v/z')/P_v) \quad (24)$$

This allows for the possibility of rational speculative overpricing or price bubble away from fundamentals.

Assuming rational expectations, the expected variation in housing prices matches that suggested by the model:

$$E(\dot{P}v/z') = \dot{P}_v \quad (25)$$

Rewriting the expression for the use cost of capital (24):

$$\varpi = rf(1 - t) + \tau + \delta - \dot{P}_v/P_v \quad (26)$$

Relating the actual equilibrium condition (23) in the rental market to the property market, with the expression (26) for the use cost of capital per unit invested:

$$R(S_v, rr, r_f, y_{dp}, x^{'}) = [rf\ (1 - t) + \tau + \delta - \dot{P}_v/P_v]\ P_v \quad (27)$$

Equation (27) indicates that expected and realized increases in real housing prices generate capital gains. And under a situation of joint equilibrium between housing and rental markets, real price increases should translate into a proportional increase in rental flows. But this is not necessarily what happens in the presence of asymmetric indexation.

By subtracting $\dot{P}_v$ in equation (27):

$$\dot{P}_v = (rf\ (1 - t) + \tau + \delta)P_v - R(S_v, rr, r_f, y_{dp}, x^{'}) \quad (28)$$

The differential equation (28) shows the dynamics of housing prices in the short run, subject to variables and parameters that determine the evolution of economic cycles and that establish the market equilibrium value of the cost use and the value of rents. In the long-run steady state, with $\dot{P}_v$ = 0, an equilibrium relationship would hold between $\dot{P}_v$ and R, for a given level of existing housing stock ($S_v = \bar{S}_v$).

The market price of new housing constructed (P_v^e) is related to the evolution of the residential housing stock ($\dot{S}_v$). The stock is given by the difference between gross investment or new housing construction (I), which depends on price and construction cost(CC)—price of land, labor, and materials—and the depreciation of the existing capital stock ($\delta\overline{S_v}$).

Thus, the differential equation for residential capital is:

$$\dot{S}_v = I(P_v, CC) - \delta S_v \tag{29}$$

Equations (28) and (29) define the short-run dynamics betweenP_v and S_v and their path to the long-run steady state. In such a state, it is assumed that there are no capital gains or losses so $\dot{P}_v$= 0. And the generation or investment in new houses only covers those that are fully depreciated, i.e., $I = \delta S_v$, so $\dot{S}_v = 0$.

$$\bar{S}_v = I(P_{v,}CC)/\delta \tag{30}$$

According to equation (23), the equilibrium price that empties the new housing market is represented by:

$$\bar{P}_v = R\left(S_v, rr, r_f, y_{dp}, x^{\prime}\right)/\varpi \tag{31}$$

Applying a linear first order Taylor approximation for equations (31) and (30), a linear relationship between price and new home sales can be established with their respective fundamentals:

$$\bar{P}_v = \frac{1}{\varpi}\left(\theta_{pvo} + \theta_{pv1}\,\bar{S}_v + \theta_{pv2}\,rr + \theta_{pv3}r_f + \theta_{pv4}\,y_{dp} + \Theta_{pv}x^{\prime} + \mu_{pv}\right) \tag{32}$$

$$\bar{S}_v = \frac{1}{\delta}\left(\theta_{so} + \theta_{s1}\,\bar{P}_v + \theta_{s2}\,CC + \mu_{sv}\right) \tag{33}$$

4.2. The SVARX model

In the context of a dynamic general equilibrium model, equations (32) and (33) can be extended to a dynamic structural system of simultaneous equations. We consider ss jointly determined endogenous variables the price of new housing ($\bar{P}_v$), rents (R), the gap between prices and rents (GAP), the sale of new housing ($\bar{S}_v$), the real interest rate of mortgage loans (rr) and the cost of construction (CC), where GAP is a non-equilibrium variable resulting from the short-run equilibrium of the model. Among the exogenous variables, we consider real disposable income (Y_{dp}), the alternative return on financial instruments (r_f), the level of mortgage indebtedness with respect to disposable income ($D_{H\_}Y_{dp}$), and the expectations of economic agents (E_x). Clearly, $D_{H\_}Y_{dp}$ and E_x can reach unbalancing values or ranges that are not sustainable in the long run. For simplicity, demographic variables are omitted because their trajectories would be rather relevant only in the very long run.

In addition, we consider the observation made by Sims (1980) that *structural models* of simultaneous equations may contain a large number of restrictions that are not very credible and may be unsupported by economic theory. Thus, the model is represented by a dynamic structural system of simultaneous equations, composed of an autoregressive vector of endogenous variables. The set of explanatory variables is specified as leading indicators, since this allows us to perform an impact/intervention analysis, based on dynamic shocks on the endogenous variables, with the only restriction of the order p lags, known as a SVARX(p) model:

$$P_{vt} = \alpha_{Pv} + \sum_{j=1}^{p} \alpha_{Pvj} P_{vt-j} + \sum_{j=0}^{p} \alpha_{Pv.Rj} R_{t-j} + \sum_{j=0}^{p} \alpha_{Pv.GAPj} GAP_{t-j} + \sum_{j=0}^{p} \alpha_{Pv.Svj} S_{vt-j} + \sum_{j=0}^{p} \alpha_{Pv.rrj} rr_{t-j} + \sum_{j=0}^{p} \alpha_{Pv.CCj} CC_{t-j} + \sum_{j=0}^{p} \beta_{Pv.Ydpj} Y_{dpt-j} + \sum_{j=0}^{p} \beta_{Pv.rfj} r_{f\,t-j} + \sum_{j=0}^{p} \beta_{Pv.DHj} D_{H\_}Y_{dp\ t-j} + \sum_{j=0}^{p} \beta_{Pv.Exj} E_{x_{t-j}} + \mu_{Pvt} \tag{34}$$

where j = 0,1,2,3 … p is the number of lags and $j = 0$ represents the contemporaneous relationship between two variables. The parameters α'_s and β'_s accompany the dynamics of the endogenous and exogenous variables in the model. The error term μ_{pvt} is usually assumed to be a white noise with zero mean and constant variance.

Extending the model for the rest of the endogenous variables:

$$R_t = \alpha_R + \sum_{j=0}^{p} \alpha_{R.Pvj} P_{vt-j} + \sum_{j=1}^{p} \alpha_{Rj} R_{t-j} + \sum_{j=0}^{p} \alpha_{R.GAPj} GAP_{t-j} + \sum_{j=0}^{p} \alpha_{R.Svj} S_{vt-j} + \sum_{j=0}^{p} \alpha_{R.rrj} rr_{t-j} + \sum_{j=0}^{p} \alpha_{R.CCj} CC_{t-j} + \sum_{j=0}^{p} \beta_{R.Ydpj} y_{dpt-j} + \sum_{j=0}^{p} \beta_{R.rfj} r_{f_{t-j}} + \sum_{j=0}^{p} \beta_{R.DHj} D_{H\_}Y_{dp_{t-j}} + \sum_{j=0}^{p} \beta_{R.Exj} E_{x_{t-j}} + \mu_{Rt} \quad (35)$$

$$GAP_t = \alpha_{GAP} + \sum_{j=0}^{p} \alpha_{GAP.Pvj} P_{vt-j} + \sum_{j=0}^{p} \alpha_{GAP.Rj} R_{t-j} + \sum_{j=1}^{p} \alpha_{GAPj} GAP_{t-j} + \sum_{j=0}^{p} \alpha_{GAP.Svj} S_{vt-j} + \sum_{j=0}^{p} \alpha_{GAP.rrj} rr_{t-j} + \sum_{j=0}^{p} \alpha_{GAP.CCj} CC_{t-j} + \sum_{j=0}^{p} \beta_{GAP.Ydpj} y_{dpt-j} + \sum_{j=0}^{p} \beta_{GAPj} r_{f_{t-j}} + \sum_{j=0}^{p} \beta_{GAP.DHj} D_{H\_}Y_{dp_{t-j}} + \sum_{j=0}^{p} \beta_{GAP.Exj} E_{x_{t-j}} + \mu_{GAPt} \quad (36)$$

$$Sv_t = \alpha_S + \sum_{j=0}^{p} \alpha_{Sv.Pvj} P_{vt-j} + \sum_{j=0}^{p} \alpha_{Sv.Rj} R_{t-j} + \sum_{j=0}^{p} \alpha_{Sv.Gapj} Gap_{t-j} + \sum_{j=1}^{p} \alpha_{Svj} S_{vt-j} + \sum_{j=0}^{p} \alpha_{Sv.rrj} rr_{t-j} + \sum_{j=0}^{p} \alpha_{Sv.CCj} CC_{t-j} + \sum_{j=0}^{p} \beta_{Sv.Ydpj} y_{dpt-j} + \sum_{j=0}^{p} \beta_{Sv.rfj} r_{f_{t-j}} + \sum_{j=0}^{p} \beta_{Sv.DHj} D_{H\_}Y_{dp_{t-j}} + \sum_{j=0}^{p} \beta_{Sv.Exj} E_{x_{t-j}} + \mu_{Svt} \quad (37)$$

$$rr_t = \alpha_{rr} + \sum_{j=0}^{p} \alpha_{rr.Pvj} P_{vt-j} + \sum_{j=0}^{p} \alpha_{rr.Rj} R_{t-j} + \sum_{j=0}^{p} \alpha_{rr.GAPj} GAP_{t-j} + \sum_{j=0}^{p} \alpha_{rr.Svj} S_{vt-j} + \sum_{j=1}^{p} \alpha_{rrj} rr_{t-j} + \sum_{j=0}^{p} \alpha_{rr.CCj} CC_{t-j} + \sum_{j=0}^{p} \beta_{rr.Ydpj} y_{dpt-j} + \sum_{j=0}^{p} \beta_{rr.rfj} r_{f_{t-j}} + \sum_{j=0}^{p} \beta_{rr.DHj} D_{H\_}Y_{dp_{t-j}} + \sum_{j=0}^{p} \beta_{rr.Exj} E_{x_{t-j}} + \mu_{rrt} \quad (38)$$

$$CC_t = \alpha_{CC} + \sum_{j=0}^{p} \alpha_{CC.pvj} P_{vt-j} + \sum_{j=0}^{p} \alpha_{CC.Rj} R_{t-j} + \sum_{j=0}^{p} \alpha_{CC.GAPj} GAP_{t-j} + \sum_{j=0}^{p} \alpha_{CC.Svj} S_{vt-j} + \sum_{j=0}^{p} \alpha_{CC.rrj} rr_{t-j} + \sum_{j=1}^{p} \alpha_{CCj} CC_{t-j} + \sum_{j=0}^{p} \beta_{CC.Ydpj} y_{dpt-j} + \sum_{j=0}^{p} \beta_{CC.rfj} r_{f_{t-j}} + \sum_{j=0}^{p} \beta_{CC.DHj} D_{H\_}Y_{dp_{t-j}} + \sum_{j=0}^{p} \beta_{CC.Exj} E_{x_{t-j}} + \mu_{CCt} \quad (39)$$

Starting from (14), we can obtain an expression for the indicator measuring the asymmetric indexation gap, which is responsible for promoting the formation of the housing price bubble "i" in each period "t". This indicator is discounted by the 30-day interest rate (r_t) and debugged by the opportunity cost of investing in the stock market $\left(r_{f_t}\right)$ coming from various productive sectors. Thus, we have:

$$GAP_i(t) = \left(\frac{[\bar{P}_{v_t} - \bar{P}_{vt-1}] - [R_t - R_{t-1}]}{[1 + r_t]^t} \; x \; [1 - (\frac{r_{f_t}}{r_{f_{t-1}}} - 1)] \right) \quad (40)$$

From (40), we can see that positive and systematic variations of the stock market index weaken the value of the bubble because they represent situations where investors have a higher opportunity cost of investing in properties. Likewise, negative systematic variations of the stock market index would stimulate investment in real estate assets, encouraging higher housing prices. This is especially relevant in a market that has been characterized in Chile by a growth in the participation of small- and medium-sized investors in home purchases. According to Castillo and López-Morales (2021), the share of home purchases for investment in Chile grew from 13 percent in 2010 to 20 percent in 2018. In the case of second homes the growth was from 17.9 percent in 2011 to 43.5 percent in 2018. Likewise, between 2010 and 2019 mortgage debt as a percentage of the country's total debt grew from 20 to 30 percent.

Considering *n* endogenous variables and *k* exogenous variables the *structural model* can be rewritten in matrix form as:

$$\boldsymbol{A}_{(\boldsymbol{nxn})}\boldsymbol{Y}_{\boldsymbol{t}\,(\boldsymbol{nx1})} = \boldsymbol{A}_{\boldsymbol{o}\,(\boldsymbol{nx1})} + \boldsymbol{Ap}_{\,(\boldsymbol{nxnp})}\boldsymbol{Y}_{\boldsymbol{t-p}\,(\boldsymbol{npx1})} + \boldsymbol{Bk}_{\,(\boldsymbol{nxk(p+1)})}\boldsymbol{X}_{\boldsymbol{t-p}\,(\boldsymbol{k(p+1)x1})} + \boldsymbol{\mu}_{\boldsymbol{t}\,(\boldsymbol{nx1})} \tag{41}$$

where matrix $\boldsymbol{B}$ contains the structural parameters of the contemporaneous and lagged exogenous variables and $\boldsymbol{X}$ is the matrix that their values. Also, $\boldsymbol{\mu}_{\boldsymbol{t}\,(\boldsymbol{nx1})}$ is presumed to be a white noise stochastic error vector, with zero mean vector and symmetric variances and covariances matrix, with stable coefficients over time, i.e., $Cov(\mu_{nt}, \mu_{lt})_{nxn} = \Omega_{nxn}$.

From the *structural* model (41), it can also be seen that all the equations share the same contemporaneous and lagged endogenous and exogenous variables, which generates identification problems, unless restrictions are included on the structural parameters limiting the presence of certain predetermined variables. We will also see that the contemporaneous endogenous variables, as explanatory variables, present correlation problems with the errors of the same equation, which induces serial autocorrelation of the errors leading to biased and inconsistent estimates.

One way to solve the above is to consider the system of simultaneous equations as an *unrestricted reduced form* SVARX system, where the contemporaneous values of the endogenous variables do not appear as explanatory variables. This can be obtained by multiplying the inverse of the parameter matrix $A_{(nxn)}$, ($A^{-1}_{(nxn)}$), which accompanies the autoregressive vector of endogenous variable$Y_{t\ (nx1)}$ on both sides of (41):

$$Y_{t\ (nx1)} = A^{-1}_{(nxn)}A_{o\ (nx1)} + A^{-1}_{(nxn)}Ap_{\ (nxnp)}Y_{t-p\ (npx1)} + A^{-1}_{(nxn)}\ Bk_{\ (nxkp)}X_{t-p\ ((kpx1)} + A^{-1}_{(nxn)}\mu_{t\ (nx1)} \tag{42}$$

Thus, the system of equations of the *unrestricted reduced form* is represented in matrix form by:

$$\boldsymbol{Y_{t\ (nx1)}} = \boldsymbol{\Gamma_{o\ (nx1)}} + \boldsymbol{\Gamma_{1\ (nxnp)}}\boldsymbol{Y_{t-p\ (npx1)}} + \mathbf{E}_{\ (\boldsymbol{nxkp})}\boldsymbol{X_{t-p\ ((kpx1)}} + \boldsymbol{\varepsilon_{t\ (nx1)}} \tag{43}$$

where $\Gamma_{o\ (nx1)} = A^{-1}_{(nxn)}A_{o\ (nx1)}$, $\Gamma_{1\ (nxnp)} = A^{-1}_{(nxn)}Ap_{\ (nxnp)}$, $\mathrm{E}_{\ (nxkp)} = A^{-1}_{(nxn)}\ Bk_{\ (nxkp)}$ and $\varepsilon_{t\ (nx1)} = A^{-1}_{(nxn)}\mu_{t\ (nx1)}$.

Here, the reduced form parameters of $\gamma_s^{\prime}$ y $\phi_s^{\prime}$, known as impact or short-run multipliers, are nonlinear combinations of structural parameters $\alpha_s^{\prime}$ y $\beta_s^{\prime}$. Likewise, the disturbance terms ε_t are a nonlinear function of the errors (μ'_{ts}) and the respective structural parameters. All this allows us to reduce problems of serial autocorrelation of the errors and multicollinearity among the explanatory variables and to achieve greater estimation efficiency.

4.3. Bayesian estimation method

The dynamic *structural model* of simultaneous equations (41) assumes that the endogenous variables in the vector $Y_{t\ (nx1)}$ are stationary and the error terms in the vector $\mu_{t\ (nx1)}$ are white noise, i.e., normally and independently distributed, with expectation E $(\mu_{n)}) = 0$ and variances $\sigma^2_{\mu n}$ stable over time (n = 1, 2, ... ,6). The contemporaneous covariances of errors of type $\sigma_{\mu n\ \mu l}$ (n, l = 1, 2, … ,6) are assumed to be time invariant, all of which is characterized by the symmetric matrix $Cov(\mu_{nt}, \mu_{lt})_{nxn} = \Omega_{nxn}$. It is also assumed that the *k* explanatory variables of the vector X_k are predetermined ensuring that they are uncorrelated with the respective error term, i.e., $E(X_{k\ t-j}\ \mu_{nt})$ = 0, $\forall_j \geq 0$.

However, from the *structural version* of model (41) an unexpected shock in μ_{Rt} in equation (35) directly affects R_t and indirectly affects P_{vt}, since it is a contemporaneous endogenous explanatory variable in equation (34). This is a problem that would be present in all equations, whereby the contemporaneous endogenous variables cease to be deterministic. Thus, the pairs of stochastic structural disturbances, by essence, will not only be contemporaneously correlated, but may contain serial correlation. Furthermore, problems of robustness, efficiency, and asymptotic consistency may arise due to the presence of collinearity between the predetermined variables, the serial and contemporaneous correlation of the errors, and the large number of parameters to be estimated.

Accordingly, Bayesian estimation allows, by means of prior knowledge of the probability density function, to incorporate restrictions on the parameters in the face of the large number of estimates in multivariate dynamic models. This improves the robustness, consistency, and efficiency of the estimated parameters. For all the above, we will consider the reduced *unrestricted version* of the model, i.e., only lagged endogenous variables ($\alpha_{yi.yl.o} = 0$) and only contemporaneous exogenous variables ($\beta_{Yi.Xrj}$ = 0), as could be seen from (43).[3] Thus, the SVARX model can be estimated based on a Bayesian full information method known as BSVARX, considering simultaneity, multicollinearity, nonlinearity, dynamic relationship, and serial correlation among the predetermined endogenous variables.

To obtain more efficient estimates, we use Markov Chain Monte Carlo (MCMC) sampling, where we assume a conjugate Minnesota a priori distribution proposed by R. B. Litterman (1986) for VAR models considering the Mean Squared Error (MSE) test of the estimators. Thus, each mean of the parameters to be estimated from the VAR ($\hat{\gamma}_s^{\prime}$ y $\hat{\phi}_s^{\prime}$) follows an a priori normal white noise distribution with an error covariance matrix ($\hat{\sigma}^2$) and a prior i-Wishart conjugate i-Wishart distribution, i.e., a multi-dimensional i-Gamma distribution. Thus, we obtain posterior distributions of the estimated mean and covariance parameters. Finally, we must consider the possible autocorrelation generated by this procedure, which, if excessive, could indicate model identification problems.

4.4. Data and variables

The real price (*in UFs*) of housing (P_v) on a quarterly basis is prepared monthly by the Chilean Chamber of Construction (CChC), based on the work of Idrovo and Lennon (2011). It is a Fischer price index, following the hedonic price methodology, based on the sales promises of new homes, houses, or apartments, of real estate developers in the city of Santiago published since January 2003. It discriminates four geographic zones and differentiates houses (P_{v_Houses})

[3] This is set by default by STATA 17.

(27%) from apartments ($P_{v\_Apartms}$) (73 percent) (Figure 1), just as the general housing price index (P_{v_Homes}) is elaborated.

The nominal index of the price of effective rents is obtained as a component of the CPI basket and is prepared monthly by NSI. It includes houses and apartments based on a sample of all the municipalities of Santiago and the regional capitals of the country. It is deflated by the CPI to obtain an index of the real value of rents (R).

A gap proxy (GAP) is constructed for houses, apartments and housing in general ($GAP_{\_Houses}$, $GAP_{\_Apartms}$, and $GAP_{\_Homes}$), considering the differences between the real housing price index (Pv) and the real rental value index (R). It is discounted by the real interest rate of the 30-day average annual of the financial system published monthly by the Central Bank of Chile (BCCh), according to the definition of equation (40), and it is corrected by the opportunity cost of investing in housing represented by the monthly stock market return $(\frac{r_{f_t}}{r_{f_{t-1}}} - 1)$.

The monthly new housing unit sales index($\bar{S}_v$) for Santiago is published monthly by the CChC since January 2003. It distinguishes apartments from houses as well as considers the total sum of both types of housing, but without distinguishing the four geographic zones of Santiago. It considers the annual moving quarter variations of units sold ($\bar{S}_{v\_Houses\_QUA}$, $\bar{S}_{v\_Apartms\_QUA}$ y $\bar{S}_{v\_Homes\_QUA}$), which would better synthesize the sales trajectory.

The real interest rate for housing mortgage loans (rr) is published monthly by the BCCh. It comes from mortgage loans granted by banks to individuals and households for the purchase of houses in UF. Currently, the largest amount is granted for three years through non-endorsable mortgage loans and mortgage bills (95 percent).

The real building or construction cost index per square meter (CC) is published monthly by the CChC. It includes the costs of materials, wages and salaries, general expenses, and site installation (UF). Since there is no public information available on the real price of land, it is assumed that the CC is the available proxy that best reflects the real construction costs (UF/m2).

The real gross national disposable income (Y_{dp}) is published quarterly by the BCCh. To convert it into monthly data, we use the Monthly Index of Economic Activity (IMACEC) published monthly by the BCCh. This is considered a high frequency indicator, following the national accounts techniques by Quilis (2001), because it is a leading indicator of the monthly GDP evolution. As expected, in the period under study, the IMACEC presents a very high quarterly simple correlation index with Y_{dp} (0.99). The annual variations of the moving quarterly Y_{dp} (Y_{dp_QUA}) are also considered, as they should better reflect the evolution of household purchasing power.

The Selective Stock Price Index is prepared by the Santiago Stock Exchange (BCS) and published by BCCh. It is a nominal indicator of the profitability of the thirty stocks with the largest stock market presence. It is deflated by the CPI to obtain the real stock market price index (rf).

The gross national disposable income commitment in personal mortgage loans($D_{H}\_Y_{dp}$) is published quarterly by the BCCh. It considers bank personal mortgage loans (D_H) as a proportion of Y_{dp}. To express it in monthly terms, we use the interpolation algorithms found in the literature for VAR models estimated with Bayesian methods (Amirizadeh (1985), Fernandez (1981), R. Litterman (1984), and R. B. Litterman (1983)).

The expectations of economic agents are collected through the Economic Perception Index(E_x) measured monthly by GFK ADIMARK Chile and published by the BCCh. Expectations about the economic situation are obtained through telephone interviews.

5. Results

Both Table 1 and Graph 1 reflect a higher growth in the trajectory of real housing prices (P_v) than in the real value of rents (R), consistent with the hypothesis of the generation of a gap (GAP) due to asymmetric indexation. Graphs 1 and 2 also confirm the absence of stationarity in the series of all variables. At glance, variables P_v, GAP, rr, Y_{dp_QUA}, rf, and E_x are governed by distributions with time-varying means and variances. Variables R, CC, and $D_{H}\_Y_{dp}$ exhibit a clear trend, reflecting temporal instability of their means. Variations in home sales $(\bar{S}_v)$ appear to be non-stationary in variance, which has been possibly accentuated by the Covid-19 period.

The augmented Dickey and Fuller (1979) test (ADF) with one and three lags (Table 2) show that it is not possible to reject the null hypothesis of unit root, or absence of stationarity, for the endogenous variables of housing prices (Pv), rents (R), mortgage interest rate(rr), and construction costs (CC)—as well as for the exogenous variables of stock market returns (rf), household debt to disposable income $\left(D_{H}\_Y_{dp}\right)$, and household expectations (Ex). The variables price gaps (GAP), home sales (Sv), and disposable income $\left(Y_{dp\_QUA}\right)$ turn out to be stationary. However, home sales $\left(\bar{S}_{v\_Houses\_QUA}\right)$ and disposable income disposable income $\left(Y_{dp\_QUA}\right)$ turn out to have three lagged unit roots (ADF).

Bartlett´s (1946) test based on the confidence bands for the autocorrelation (AC) and partial autocorrelation functions (Graph 3) indicate that housing prices, like most of the other variables in the model, follow autoregressive (AR) processes. However, price and rent gaps are identified with ARMA processes and only disposable income with a moving average (MA) process.

Despite the non-stationary stochastic behaviors, Engle and Granger (1987) indicate that cointegration between non-stationary variables, even with different level of integration, is achieved when linear -and non-linear- combinations lead to long-run stationary situations. Therefore, the BSVARX model is estimated using adaptive Markov Chain Monte Carlo (MCMC) sampling to obtain the posterior distributions of the estimated parameter matrix to analyze efficiency, convergence, and stability.

Table 3 presents a summary of the lag order selection criteria for the BSVARX model. According to Lütkepohl (1985), Schwarz's Bayesian Information Criterion (SBIC) would more frequently choose the appropriate autoregressive order considering the lowest mean square error (MSE) in relation to other criteria such as the Likelihood Ratio Test (LR), the Final Prediction Error (FPE), Akaike's Information Criterion (AIC) and the Hannan and Quin Information Criterion (HQIC). Therefore, and for homogeneity of the estimations, the models of the three housing prices are run with one lag for the endogenous variables, even though SBIC suggests two lags for the model of apartment prices.

Table 4 presents the BSVARX estimation for the equations of the six endogenous variables for the general level of housing prices (P_{v_Homes}). The MCMC sample size is 10,000 iterations and the "Burn-in" is 2,500. The acceptance ratio of the estimated parameters is high at 1.0, which could indicate a concentration of iterations in the high part of the distribution. However, the average efficiency indicator is 0.9968, indicating that there are no autocorrelation problems and possibly no model specification problems. The Max Gelman-Rubin Rc test of 1.0 indicates that convergence is achieved for all parameters of the multi-chain Markov model. Also, the stability test of the estimated parameters indicates that the probability that the estimated and decreasing eigenvalues of the model fall on the unit circle is 0.7244. The results in Tables 5 and 6 of the same BSVARX model for the price of houses (P_{v_Houses}) and apartments ($P_{v\_Apartms}$) are consistent with the above.

From the estimation of BSVARX for the home price equation (Table 4), all the means of the estimated coefficients have the expected sign, except for Y_{dp_QUA}, which is statistically non-significant, as is R although with the expected negative sign. The mean of the estimated coefficient accompanying $GAP_{\_Homes}$ is 0.2484 and statistically significant, which could indicate, consistent with our working hypothesis, that the risk aversion of investing in properties could be accumulating over time an average overprice gap, as a proxy to a rational speculative price bubble, equivalent to 24.8 percent of the gap formed between prices and rents. Tables 5 and 6 show that the average of the estimated average coefficient for the gap for houses is 0.1599, lower than the average gap coefficient 0.2074 for apartments; it is this latter where the largest number of investors tend to concentrate—the ones who tend to speculate more on prices.

Graphs 4, 5, 7, and 9 show traces defining a stationary sequence of estimates for the estimated gap parameters supporting convergence. The autocorrelation functions oscillate randomly in a range of 0.02 to -0.03 reflecting the possible efficiency of the MCMC, despite a possible concentration of values for the gap coefficient. Both histograms and posterior density functions validate the assumption of normality of these coefficients, ruling out bimodal densities despite what is observed in the histograms, with means and standard deviations shown in the tables.

The impulse response functions (IRF) shown in Graphs 6, 8, and 10 describe the reaction of endogenous variables, particularly in house prices, to the shock and at subsequent times of other variables, whether endogenous or exogenous. The results indicate that the impact of one month's GAP does not dissipate easily and is rather permanent on housing prices in the following eight months. This behavior is even more relevant for the price of apartments, where, as mentioned above, a greater participation of small and medium investors is presumed. In the case of houses, the downward behavior is more pronounced, although it does not disappear in the eight months considered. All the above validates our working hypothesis on the impact of these price gaps on the price of housing of all types, which is mainly a phenomenon of a continuous and permanent character rather than a discrete one.

For the general housing price index (Table 7), the gap variable grows by 88.213 points over the entire period, which, cleaned by the coefficient estimated in our model $\left(\hat{\gamma}_{pv.GAP}\right)$ of 0.2484, yields a price premium of 21.9 points. Given that housing prices grew over the entire period by 131 points, the price premium for this item would be 16.7 percent. Considering the impulse response functions we can assume, by means of a geometric progression, that this rate as a proxy to the price bubble could well reach 20.1 percent. Similarly, in the case of houses, the highest price growth or bubble would oscillate between 10.5 and 11.8 percent and in apartments between 14.1 and 16.4 percent, justified by the possible greater presence of investors in the apartment market.

To measure the levels of endogeneity of the BSVARX model, in addition to the three equations already analyzed from Tables 4, 5, and 6, we also consider the other 15 estimated equations of the model that are presented in the same tables. Both at the general level of the housing market and for the submarkets of houses and apartments, we find a higher level of endogeneity of the price variable (Pv) with rents (R), gaps (GAP), and home sales (Sv), but not with the mortgage interest rate (rr) and construction costs (CC). Likewise, explanatory variables such as stock market returns (rf), household debt to disposable income $\left(D_{H}\_Y_{dp}\right)$, and household expectations (Ex) turn out to be statistically significant in the estimated equations at the three price levels for rents (R), gaps (GAP), and home sales (Sv). However, they are not statistically significant in explaining the behavior of the mortgage interest rate (rr) and construction costs (CC).

6. Conclusions

This paper uses a general equilibrium model for the housing market in Santiago of Chile over the last 20 years, which validates the hypothesis that risk aversion generated by an asymmetric indexation between housing prices and mortgages in relation to rental values produces overpricing, which can be conceptualized as rational speculative price bubbles. This situation would be even more relevant in the apartment market than in the housing market due to a greater presence of small and medium investors in the former.

This finding is in line with the work of Miles and Pillonca (2008) that highlights the speculative role played by rising expectations of future capital appreciation gains of homebuyers in Spain, Sweden, Belgium, and the United Kingdom. The asymmetry between prices and inflation-indexed mortgages and rents could lead to house prices beyond what fundamentals support, i.e., price bubbles. It is also in line with Campbell and Cocco (2003) who conclude that, while indexing house prices and mortgages would provide inflation insurance for households and investors, it could also encourage speculative behavior in the expected future path of real house prices.

It is evident that among the benefits of indexation through the UF in Chile would be the deepening of the capital market through the incentives that come with risk-protected savings and inflationary erosion, translated into higher investment rates, greater economic growth, and higher employment levels (Fontaine 2002). However, among the obvious costs would be the eventual loss of effectiveness of monetary policy in controlling inflation due to the inertia it generates (Fischer 1983), in addition to the changes in the relative price structure that a partial indexation would entail that would affect an optimal and efficient allocation of resources, with the obvious welfare costs. And, finally, costs not studied so far, such as the possible formation of rational speculative price bubbles due to risk premiums demanded under asymmetric price indexation processes.

As future developments, it would be desirable to study and quantify the social cost of the population that is excluded from acquiring a home due to these higher prices detected in Europe and Chile. It is also necessary to discuss the role that the economic authority should play in the face of policies, regulations, and practices of asymmetric price indexation between sectors and highly integrated markets. It seems essential for the economic authority to identify and monitor these types of situations and practices to manage the price distortions they may cause. Applying corrections and regulations along the lines of avoiding them or extending them to all related sectors. It would also be interesting to analyze in more detail how the various speculative housing price behaviors of investors, due to asymmetric price indexation practices, influence the formation of these price bubbles.

Finally, there is the possibility of extending the study of asymmetric price indexation and its impact in other areas such as labor, education, health, taxation, services, and the external sectors—areas where asymmetric price indexation is commonly used in various countries, often introduced by the economic authority itself.

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Tables

Table 1: Descriptive Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Time	234	117.5	67.694	1	234
Date	234	19251.397	2060.459	15706	22797
Pv_Houses	234	140.602	39.335	96.429	234.103
Pv_Apartms	234	142.759	42.152	96.802	230.073
Pv_Homes	234	142.107	41.303	97.496	230.985
R	234	107.063	6.986	94.942	118.664
GAP_Houses	234	0.378	1.525	-3.933	5.517
GAP_Apartms	234	0.379	1.312	-3.044	5.508
GAP_Homes	234	0.377	1.090	-3.129	3.877
Sv_Houses_QUA	234	0.944	0.340	0.000	1.927
Sv_Apartms_QUA	234	1.015	0.420	0.000	3.237
Sv_Homes_QUA	234	0.985	0.365	0.000	2.490
rr	234	4.020	0.813	1.990	5.920
CC	234	180.835	53.037	100.000	292.335
Ydp_QUA	234	0.977	0.246	0.000	1.291
rf	234	252.391	65.278	98.754	384.090
DH_ Ydp	234	30.683	7.998	16.280	46.930
Ex	234	104.580	28.015	30.506	145.918
DUMMY_Ydp	234	0.128	0.335	0	1

Table 2: Augmented Dickey-Fuller Tests (DF and DFA) for unit root

Variables	Z(t)	p-value	lags	Z(t)	p-value	lags
Pv_Houses	-1.401	0.861	1	-0.796	0.966	3
Pv_Apartms	-1.999	0.602	1	-1.908	0.650	3
Pv_Homes	-1.739	0.733	1	-1.583	0.799	3
R	-1.693	0.754	1	-1.513	0.825	3
GAP_Houses	-9.984	0.000	1	-7.818	0.000	3
GAP_Apartms	-9.308	0.000	1	-7.895	0.000	3
GAP_Homes	-9.412	0.000	1	-7.049	0.000	3
Sv_Houses_QUA	-4.872	0.000	1	-3.088	0.109	3
Sv_Apartms_QUA	-6.129	0.000	1	-4.691	0.001	3
Sv_Homes_QUA	-5.426	0.000	1	-4.598	0.001	3
rr	-3.348	0.059	1	-2.885	0.167	3
CC	-1.577	0.801	1	-2.323	0.421	3
Ydp_QUA	-5.734	0.000	1	-3.329	0.062	3
rf	-1.765	0.721	1	-1.691	0.755	3
DH_ Ydp	-1.772	0.718	1	-2.340	0.412	3
Ex	-2.861	0.176	1	-2.576	0.291	3

Dickey-Fuller Z(t)

	1%	5%	10%
Critical Value	-3.997	-3.433	-3.133

Table 3: Lutkepohl's Lag-order Selection Criteria

Sample: 2005m3 thru 2022m6 Number of obs = 208

Pv_Homes

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-4711.49	6248.8	100	0.000	2.50E+07	16.924	16.924	16.924
1	-1587.08	507.26	100	0.000	5.80E-06	-12.157	-11.508	**-10.5522***

Pv_Houses

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-4780.03				4.80E+07	17.583	17.583	17.583
1	-1838.08	5883.9	100	0.0000	0.000065	-9.74341	-9.0946	**-8.13883***

Pv_Apartms

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-4815.14				6.70E+07	17.9206	17.9206	17.9206
1	-1744.83	6140.6	100	0.000	0.000026	-10.64	-9.9912	-9.03543
2	-1472.16	545.35	100	0.000	5.00E-06	-12.3003	-11.0027*	**-9.09118***

Table 4: Bayes VAR Model for Pv_Homes Equation

Bayesian vector autoregression	MCMC iterations		12,500
Gibbs sampling	Burn-in		2,500
	MCMC sample size		10,000
Sample: 2004m4 thru 2022m6	Number of obs		219
	Acceptance rate		1
	Efficiency:	min	0.9456
		avg	0.9968
Log marginal-likelihood = -1294.6319		max	1
	Max Gelman–Rubin Rc		1.0

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]	
Pv_Homes						
Pv_Homes						
L1.	0.9418438	0.0137879	0.000138	0.9420392	0.9148556	0.9686573
R						
L1.	-0.0294201	0.04227	0.000423	-0.0297451	-0.1114156	0.0541054
GAP_Homes						
L1.	0.2484346	0.0673881	0.000674	0.2489853	0.1170049	0.3788924
Sv_Homes_QUA						
L1.	0.752578	0.3453771	0.003454	0.7522254	0.0829754	1.437167
rr						
L1.	-0.5482632	0.2198096	0.002159	-0.5471051	-0.9769289	-0.1206739
CC						
L1.	0.0266572	0.0100151	0.000101	0.0266637	0.007013	0.0460251
Ydp_QUA	-2.800869	1.704441	0.017044	-2.801648	-6.141717	0.5915987
rf	-0.009326	0.0030738	0.00003	-0.009316	-0.0153499	-0.0033067
DH_Ydp	0.2354915	0.0875032	0.000888	0.2349497	0.0642457	0.4085879
Ex	0.0263204	0.0072264	0.000072	0.0262483	0.0122661	0.0407407
DUMMY_Ydp	-0.0037469	0.0024893	0.000025	-0.0037704	-0.0085273	0.0012248
_cons	3.758879	4.090841	0.040908	3.760321	-4.081088	11.76867
R						
Pv_Homes						
L1.	-0.0181652	0.0069032	0.000069	-0.0181011	-0.0318982	-0.0045784
R						
L1.	0.9727959	0.0213074	0.000209	0.9727851	0.9302514	1.015309
GAP_Homes						
L1.	-0.057101	0.0336569	0.000337	-0.0572691	-0.1238914	0.0096764
Sv_Homes_QUA						
L1.	-0.0893514	0.1731325	0.001731	-0.0879491	-0.4295356	0.250473
rr						
L1.	-0.15864	0.1116321	0.001116	-0.1581805	-0.3795521	0.0585025
CC						
L1.	0.0028439	0.0050827	0.00005	0.0027963	-0.006995	0.0127445
Ydp_QUA	0.093204	0.8715447	0.008715	0.0930119	-1.650617	1.796036
rf	-0.0031484	0.0015459	0.000015	-0.0031436	-0.0062224	-0.0001658
DH_Ydp	0.1283892	0.0444139	0.000444	0.1286986	0.0406823	0.21467
Ex	0.0078109	0.0036727	0.000036	0.0078538	0.0005632	0.0148704
DUMMY_Ydp	-0.0037785	0.0012549	0.000013	-0.0037739	-0.0062203	-0.0013413

_cons	1.770012	2.063001	0.020413	1.760502	-2.310326	5.853432
GAP_Homes						
Pv_Homes						
L1.	-0.030342	0.012053	0.000121	-0.0304022	-0.0537072	-0.0068903
R						
L1.	0.0293481	0.0365566	0.000366	0.0286841	-0.041856	0.1017772
GAP_Homes						
L1.	0.5312062	0.0583801	0.000583	0.5316573	0.4165257	0.6466916
Sv_Homes_QUA						
L1.	0.374774	0.29851	0.002985	0.3737684	-0.2019128	0.9684547
rr						
L1.	-0.2436925	0.19201	0.00192	-0.2465906	-0.6187434	0.1424249
CC						
L1.	0.0091686	0.0087768	0.000088	0.0090973	-0.0075245	0.0264148
Ydp_QUA	-2.298836	1.487766	0.014878	-2.301269	-5.18724	0.6159062
rf	-0.0053025	0.0026796	0.000027	-0.0053129	-0.010581	-0.0000924
DH_Ydp	0.1059277	0.0766675	0.000782	0.1072718	-0.0448967	0.2568582
Ex	0.0128151	0.0063049	0.000064	0.012781	0.0003487	0.0250357
DUMMY_Ydp	-0.0009916	0.0021733	0.000022	-0.0010024	-0.0052112	0.0032469
_cons	-0.6048323	3.557913	0.035579	-0.6049632	-7.560492	6.279642
Sv_Homes_QUA						
Pv_Homes						
L1.	-0.0036859	0.0012008	0.000012	-0.0036705	-0.006018	-0.001338
R						
L1.	-0.0008203	0.0036597	0.000037	-0.0007825	-0.0080963	0.0062776
GAP_Homes						
L1.	0.0078465	0.0058495	0.000058	0.0078606	-0.0038628	0.0191764
Sv_Homes_QUA						
L1.	0.903391	0.029994	0.0003	0.9031769	0.8448107	0.9622588
rr						
L1.	-0.0403059	0.0191933	0.000195	-0.040121	-0.0783596	-0.0031699
CC						
L1.	-0.0002619	0.0008747	8.70E-06	-0.0002672	-0.0019624	0.0014262
Ydp_QUA	0.0088928	0.1510388	0.001479	0.0072354	-0.2889504	0.3055089
rf	-0.0006031	0.0002692	2.70E-06	-0.0006038	-0.0011197	-0.0000786
DH_Ydp	0.0243074	0.0076812	0.000077	0.0243638	0.0089732	0.0391137
Ex	0.0017247	0.0006379	6.20E-06	0.001719	0.0004616	0.0029651
DUMMY_Ydp	-0.0001279	0.000216	2.20E-06	-0.0001281	-0.0005504	0.0002977
_cons	0.1291674	0.3573255	0.003573	0.1245276	-0.5533377	0.8337537
rr						
Pv_Homes						
L1.	-0.0016604	0.0016098	0.000016	-0.0016592	-0.0047929	0.0014693
R						
L1.	-0.0200812	0.0049529	0.00005	-0.0200267	-0.0298494	-0.0103719
GAP_Homes						
L1.	-0.0067946	0.0078376	0.000079	-0.0068507	-0.0221374	0.0088243
Sv_Homes_QUA						
L1.	0.072773	0.0408637	0.000409	0.0724525	-0.0076191	0.1534608

rr						
L1.	0.9319697	0.0261144	0.000261	0.9318319	0.8814427	0.9823528
CC						
L1.	0.0016723	0.0011815	0.000012	0.001665	-0.0006037	0.0040401
Ydp_QUA	-0.1629577	0.2035102	0.002035	-0.15963	-0.5592048	0.2367155
rf	0.000046	0.0003633	3.60E-06	0.0000468	-0.0006655	0.0007548
DH_Ydp	0.0118268	0.0101837	0.000102	0.0116757	-0.0078917	0.0322063
Ex	0.0005518	0.0008461	8.50E-06	0.0005461	-0.001114	0.0022224
DUMMY_Ydp	0.0000995	0.0002941	2.90E-06	0.0001009	-0.0004864	0.0006686
_cons	2.005182	0.4803694	0.004804	2.001249	1.061298	2.945835
CC						
Pv_Homes						
L1.	-0.0036269	0.0243821	0.000244	-0.003793	-0.0516603	0.0438252
R						
L1.	-0.0240524	0.0752134	0.000752	-0.0240404	-0.1713446	0.1256885
GAP_Homes						
L1.	0.0811964	0.1198142	0.001198	0.0801853	-0.1511391	0.318768
Sv_Homes_QUA						
L1.	0.7848814	0.610806	0.006191	0.7714538	-0.4061218	2.004898
rr						
L1.	0.1781277	0.3937005	0.003937	0.1749691	-0.6034096	0.9478735
CC						
L1.	0.9661845	0.0179119	0.000179	0.9660992	0.9315002	1.001181
Ydp_QUA	-3.531609	3.068345	0.031088	-3.510606	-9.516651	2.52868
rf	-0.0013003	0.0055378	0.000055	-0.0012564	-0.0119922	0.0095308
DH_Ydp	0.327356	0.1573608	0.001574	0.3270148	0.0195542	0.639166
Ex	0.0121505	0.0130604	0.000131	0.0122108	-0.0135865	0.0380154
DUMMY_Ydp	-0.0009799	0.0045161	0.000045	-0.0009622	-0.0097578	0.007843
_cons	1.077234	7.31579	0.073158	1.163527	-13.43916	15.38787

Eigenvalue stability condition BVARSX for Pv_Homes

Pr(eigenvalues lie inside the unit circle) = 0.7244

Table 5: Bayes VAR Model for Pv_Houses Equation

Bayesian vector autoregression	MCMC iterations		12,500
Gibbs sampling	Burn-in		2,500
	MCMC sample size		10,000
Sample: 2004m4 thru 2022m6	Number of obs		219
	Acceptance rate		1
	Efficiency:	min	0.8718
		avg	0.9952
Log marginal-likelihood = -1514.7035		max	1
	Max Gelman–Rubin Rc		1.0

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]	
Pv_Houses						
Pv_Houses						
L1.	0.9092142	0.0204974	0.000205	0.9095041	0.8689865	0.9492093
R						
L1.	-0.1427512	0.0678299	0.000678	-0.1432432	-0.2745419	-0.0080945
GAP_Houses						
L1.	0.1598878	0.0742386	0.000742	0.1607026	0.014849	0.3034266
Sv_Houses_QUA						
L1.	1.358132	0.5533626	0.005534	1.357974	0.2740572	2.453687
rr						
L1.	-0.4649046	0.3267649	0.00318	-0.4648206	-1.110891	0.1749791
CC						
L1.	0.049254	0.015868	0.00016	0.0493518	0.0182342	0.0799934
Ydp_QUA	1.132364	2.439854	0.024399	1.111443	-3.676884	5.922947
rf	-0.0181141	0.0045396	0.000045	0.0181213	-0.0270772	-0.0092462
DH_Ydp	0.4078566	0.1334188	0.001352	0.4066565	0.146229	0.6713486
Ex	0.0330213	0.0110214	0.00011	0.0329905	0.0115593	0.0550475
DUMMY_Ydp	-0.0050908	0.0038439	0.000038	-0.0050794	-0.0125591	0.0025815
_cons	7.62052	6.653221	0.066532	7.594117	-5.540261	20.73976
R						
Pv_Houses						
L1.	-0.0212887	0.0064147	0.000064	-0.0212954	-0.0339299	-0.008714
R						
L1.	0.9640725	0.0214574	0.000211	0.9642882	0.9216711	1.006647
GAP_Houses						
L1.	-0.0366384	0.0231933	0.000232	-0.0367084	-0.0825174	0.0095651
Sv_Houses_QUA						
L1.	-0.1276203	0.1741084	0.001741	-0.1257112	-0.4719588	0.2123879
rr						
L1.	-0.1560594	0.1033286	0.001033	-0.1560059	-0.3615067	0.0456936
CC						
L1.	0.0042817	0.0050387	0.00005	0.0042654	-0.0055062	0.0141335
Ydp_QUA	0.0236854	0.7785416	0.007785	0.0198666	-1.518947	1.536181
rf	-0.0030522	0.0014272	0.000014	-0.0030532	-0.0059055	-0.0002808
DH_Ydp	0.1309124	0.0423776	0.000424	0.1314138	0.0471971	0.2134399
Ex	0.0080084	0.0035031	0.000034	0.0080316	0.0011019	0.0147447

DUMMY_Ydp	-0.0029259	0.0012098	0.000012	-0.0029141	-0.0052937	-0.0005786
_cons	2.798695	2.100836	0.021008	2.786847	-1.309288	7.001718
GAP_Houses						
Pv_Houses						
L1.	-0.0542475	0.016921	0.000169	-0.0543275	-0.0869797	-0.0210592
R						
L1.	-0.0394721	0.0554513	0.000555	-0.0404293	-0.1468524	0.0703092
GAP_Houses						
L1.	0.4545666	0.0607785	0.000607	0.4546612	0.3332835	0.5746132
Sv_Houses_QUA						
L1.	0.9071087	0.4521736	0.004522	0.9037877	0.031773	1.805422
rr						
L1.	-0.2247695	0.2686942	0.002687	-0.2264709	-0.7477473	0.3040543
CC						
L1.	0.025919	0.0131274	0.000131	0.0259022	0.0007949	0.051619
Ydp_QUA	-0.7267225	2.011727	0.020117	-0.7202507	-4.699101	3.223277
rf	-0.0113179	0.0037492	0.000037	-0.0113099	-0.0187177	-0.0039759
DH_Ydp	0.2127574	0.1104966	0.001128	0.214569	-0.0052825	0.4290946
Ex	0.0196136	0.0090843	0.000092	0.0195674	0.0015697	0.0372271
DUMMY_Ydp	-0.0017761	0.0031667	0.000032	-0.0018117	-0.0079651	0.0044297
_cons	2.397188	5.46845	0.054684	2.381577	-8.453424	12.93721
Sv_Houses_QUA						
Pv_Houses						
L1.	-0.0028804	0.001249	0.000012	-0.0028766	-0.0053063	-0.0004363
R						
L1.	-0.0000959	0.0041173	0.000041	-0.0000713	-0.0082552	0.0077863
GAP_Houses						
L1.	-0.0015208	0.004502	0.000045	-0.0015089	-0.0104577	0.0071881
Sv_Houses_QUA						
L1.	0.8669062	0.0338159	0.000338	0.8668309	0.8014299	0.9333526
rr						
L1.	-0.0434112	0.0198884	0.000202	-0.0433906	-0.0824892	-0.0044923
CC						
L1.	-0.000271	0.0009706	9.70E-06	-0.0002746	-0.0021739	0.0015932
Ydp_QUA	-0.1190305	0.1511527	0.001512	-0.1188543	-0.4127943	0.1776813
rf	-0.0002516	0.0002792	2.80E-06	-0.0002545	-0.0007859	0.0002978
DH_Ydp	0.0155936	0.0081987	0.000082	0.0156034	-0.0007458	0.0315802
Ex	0.0018056	0.000681	6.60E-06	0.0018033	0.0004763	0.0031349
DUMMY_Ydp	0.0001391	0.0002335	2.30E-06	0.0001395	-0.0003183	0.0006042
_cons	0.2795904	0.4059549	0.00406	0.2752233	-0.4994158	1.087403
rr						
Pv_Houses						
L1.	-0.0011127	0.0015244	0.000015	-0.0011129	-0.0040984	0.0018726
R						
L1.	-0.021505	0.005075	0.000051	-0.0214967	-0.0315859	-0.0115276
GAP_Houses						
L1.	-0.0024403	0.0054985	0.000055	-0.0025361	-0.0131154	0.008565
Sv_Houses_QUA						

L1.	0.0425238	0.0417711	0.000418	0.0423949	-0.0403774	0.1246562
rr						
L1.	0.9458638	0.0246604	0.000247	0.9459887	0.8979368	0.9932306
CC						
L1.	0.0017359	0.0011916	0.000012	0.0017259	-0.0005586	0.0041158
Ydp_QUA	-0.0047821	0.1856683	0.001835	-0.0021981	-0.3659839	0.3614086
rf	0.0000544	0.0003427	3.40E-06	0.0000564	-0.0006193	0.0007286
DH_Ydp	0.010645	0.0098955	0.000099	0.01051	-0.0085803	0.0303813
Ex	0.000428	0.0008227	8.20E-06	0.0004297	-0.0011984	0.0020582
DUMMY_Ydp	0.0001362	0.0002893	2.90E-06	0.0001368	-0.0004519	0.0006899
_cons	1.926296	0.4980378	0.00498	1.925514	0.9379621	2.903912
CC						
Pv_Houses						
L1.	-0.0047407	0.0229957	0.00023	-0.0048674	-0.0500804	0.0402001
R						
L1.	-0.043488	0.0768818	0.000769	-0.0437501	-0.1937317	0.1088198
GAP_Houses						
L1.	0.0742787	0.083788	0.000838	0.0732752	-0.0871405	0.2413724
Sv_Houses_QUA						
L1.	0.2692961	0.6240308	0.006344	0.2608985	-0.9343249	1.521243
rr						
L1.	0.2111633	0.3716025	0.003716	0.2120052	-0.5242896	0.9345452
CC						
L1.	0.9696253	0.0179942	0.00018	0.9694385	0.9347345	1.00493
Ydp_QUA	-2.169416	2.785173	0.028289	-2.187367	-7.489265	3.37909
rf	-0.0017089	0.0051816	0.000052	-0.0017105	-0.011859	0.0084386
DH_Ydp	0.3382044	0.1522768	0.001523	0.3383357	0.042271	0.6383504
Ex	0.0137461	0.0126345	0.000126	0.0137808	-0.0112224	0.0385734
DUMMY_Ydp	-0.000701	0.0044272	0.000047	-0.00071	-0.009333	0.007997
_cons	1.288836	7.542358	0.075424	1.415229	-13.66978	16.00764

Eigenvalue stability condition BVARSX for Pv_Houses

Pr(eigenvalues lie inside the unit circle) = 0.5238

Table 6: Bayes VAR Model for Pv_Apartms Equation

Bayesian vector autoregression	MCMC iterations	12,500
Gibbs sampling	Burn-in	2,500
	MCMC sample size	10,000
Sample: 2004m4 thru 2022m6	Number of obs	219
	Acceptance rate	1
	Efficiency: min	0.9211
	avg	0.9965
Log marginal-likelihood = -1461.7861	max	1
	Max Gelman–Rubin Rc	1.0

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]	
Pv_Apartms						
Pv_Apartms						
L1.	0.9373361	0.0170986	0.000171	0.9375777	9037637	0.9706389
R						
L1.	-0.0139926	0.0549745	0.00055	-0.0143694	-0.1199755	0.094802
GAP_Apartms						
L1.	0.2074213	0.0708994	0.000709	0.2082605	0.0686929	0.3447497
Sv_Apartms_QUA						
L1.	0.5578442	0.3633996	0.003634	0.5574328	-0.1456598	1.280801
rr						
L1.	-0.6911581	0.2882167	0.002828	-0.6892122	-1.253722	-0.1256832
CC						
L1.	0.0270601	0.0129039	0.000131	0.0270748	0.0017697	0.0521342
Ydp_QUA	-4.146716	2.284493	0.022845	-4.132776	-8.598148	0.4257748
rf	-0.0088814	0.0040787	0.00004	0.0088783	-0.0169009	-0.0008885
DH_Ydp	0.23662	0.112673	0.001144	0.2352841	0.0166174	0.4586108
Ex	0.0289776	0.0093229	0.000093	0.0289028	0.0108039	0.0475188
DUMMY_Ydp	-0.0038748	0.0033007	0.000033	-0.0039129	-0.0102272	0.0026556
_cons	4.437935	5.265387	0.052654	4.457857	-5.677387	14.79072
R						
Pv_Apartms						
L1.	-0.0132912	0.0066652	0.000067	-0.0132458	-0.026608	-0.0001658
R						
L1.	0.9809018	0.0215466	0.000212	0.9809601	0.9381324	1.023802
GAP_Apartms						
L1.	-0.0233854	0.0275813	0.000276	-0.0234061	-0.0781201	0.0314012
Sv_Apartms_QUA						
L1.	-0.0231282	0.1417675	0.001418	-0.0218368	-0.3011237	0.2548849
rr						
L1.	-0.1159249	0.1139399	0.001139	-0.1153957	-0.3414946	0.1074646
CC						
L1.	0.0000771	0.0050999	0.00005	0.0000328	-0.0097556	0.0099581
Ydp_QUA	0.0282189	0.9087097	0.009087	0.0296754	-1.773876	1.816426
rf	-0.002625	0.0015979	0.000016	-0.0026269	-0.0058016	0.0004819
DH_Ydp	0.113239	0.0445532	0.000446	0.1135953	0.0252357	0.2003319
Ex	0.0063159	0.0036913	0.000036	0.0063613	-0.0009748	0.0134407

DUMMY_Ydp	-0.0038459	0.0012974	0.000013	-0.0038422	-0.0063901	-0.0013366
_cons	1.032431	2.063516	0.020323	1.027684	-3.05899	5.132241
GAP_Apartms						
Pv_Apartms						
L1.	-0.0389145	0.014538	0.000145	-0.0389309	-0.0669837	-0.0103643
R						
L1.	0.0319907	0.0462231	0.000462	0.0315065	-0.0585189	0.1229072
GAP_Apartms						
L1.	0.4916538	0.0598036	0.000598	0.4919301	0.3736973	0.6095497
Sv_Apartms_QUA						
L1.	0.1732681	0.3060436	0.00306	0.1717072	-0.4133637	0.7789474
rr						
L1.	-0.3665696	0.2447141	0.002447	-0.3708751	-0.8482877	0.1221863
CC						
L1.	0.0131517	0.0110055	0.00011	0.013049	-0.0077179	0.0347249
Ydp_QUA	-2.411886	1.936822	0.019368	-2.409656	-6.222301	1.404894
rf	-0.0061398	0.003458	0.000035	-0.0061484	-0.0129075	0.0005621
DH_Ydp	0.1262298	0.0960747	0.000982	0.1277212	-0.0630476	0.3156578
Ex	0.01503	0.0079127	0.000081	0.0149489	-0.0006851	0.0303265
DUMMY_Ydp	-0.0014965	0.0028011	0.000028	-0.0015126	-0.0069244	0.004015
_cons	-0.1823613	4.445083	0.044451	-0.1996041	-8.941515	8.426175
Sv_Apartms_QUA						
Pv_Apartms						
L1.	-0.0032671	0.0015123	0.000015	-0.0032527	-0.0062081	-0.0003264
R						
L1.	-0.0014716	0.0048357	0.000048	-0.0014347	-0.011118	0.0078773
GAP_Apartms						
L1.	0.0124291	0.006246	0.000062	0.012451	-0.0000802	0.024584
Sv_Apartms_QUA						
L1.	0.8897203	0.0320195	0.00032	0.8895326	0.827443	0.9532051
rr						
L1.	-0.035206	0.0255928	0.00026	-0.0350581	-0.0858531	0.0146793
CC						
L1.	-0.0004375	0.0011446	0.000011	-0.0004381	-0.0026769	0.0017707
Ydp_QUA	0.159671	0.2054491	0.002006	0.1570454	-0.2474517	0.5640448
rf	-0.0006419	0.0003623	3.60E-06	-0.0006451	-0.0013333	0.0000651
DH_Ydp	0.0250941	0.0100448	0.0001	0.0251395	0.005065	0.0445357
Ex	0.0013329	0.0008361	8.10E-06	0.001331	-0.0003015	0.0029626
DUMMY_Ydp	-0.0002286	0.0002905	2.90E-06	-0.0002281	-0.0007942	0.0003517
_cons	0.0425569	0.4676352	0.004676	0.0376175	-0.8526665	0.9698259
rr						
Pv_Apartms						
L1.	-0.0015424	0.0015353	0.000015	-0.0015423	-0.0045212	0.001449
R						
L1.	-0.0195306	0.004954	0.00005	-0.0194741	-0.0292748	-0.0098474
GAP_Apartms						
L1.	-0.0048116	0.0063408	0.000064	-0.0048718	-0.0171252	0.007843
Sv_Apartms_QUA						

L1.	0.0531683	0.0330744	0.000331	0.0530079	-0.0117169	0.1180302
rr						
L1.	0.9299454	0.0263573	0.000263	0.9298096	0.8784719	0.9809684
CC						
L1.	0.001553	0.0011709	0.000012	0.0015466	-0.0007155	0.0038898
Ydp_QUA	-0.1732634	0.2097231	0.002097	-0.1699822	-0.5803257	0.2399196
rf	0.0000872	0.0003706	3.70E-06	0.0000885	-0.0006366	0.0008119
DH_Ydp	0.0112547	0.0100907	0.000101	0.0111391	-0.0083584	0.0314777
Ex	0.0005816	0.0008397	8.40E-06	0.0005774	-0.0010691	0.002228
DUMMY_Ydp	0.0001122	0.0003001	3.00E-06	0.0001139	-0.0004835	0.0006953
_cons	1.99246	0.475279	0.004753	1.990819	1.056996	2.922948
CC						
Pv_Apartms						
L1.	-0.0010971	0.0232294	0.000232	-0.0012304	-0.0469059	0.0439017
R						
L1.	-0.0197995	0.0750207	0.00075	-0.0199438	-0.1672859	0.129177
GAP_Apartms						
L1.	0.0489807	0.0968411	0.000968	0.0485245	-0.1377915	0.2403857
Sv_Apartms_QUA						
L1.	0.7074811	0.4935062	0.005002	0.6988417	-0.2542389	1.692998
rr						
L1.	0.1574313	0.3960981	0.003961	0.1546751	-0.6237567	0.9309366
CC						
L1.	0.9648148	0.0177415	0.000177	0.9647647	0.9302445	0.9993153
Ydp_QUA	-3.922847	3.158762	0.031969	-3.889149	-10.11887	2.275117
rf	-0.0002911	0.0056511	0.000057	-0.0002876	-0.0112774	0.0107705
DH_Ydp	0.310583	0.1558106	0.001558	0.3099658	0.0064334	0.6195142
Ex	0.0118411	0.0129435	0.000129	0.0118855	-0.0136235	0.0374338
DUMMY_Ydp	-0.000452	0.0046	0.000046	-0.0004389	-0.0094093	0.0085385
_cons	1.347856	7.228254	0.072283	1.369377	-12.76965	15.44922

Eigenvalue stability condition BVARSX for Pv_Apartms

Pr(eigenvalues lie inside the unit circle) = 0.6618

Table 7: Bubble determination

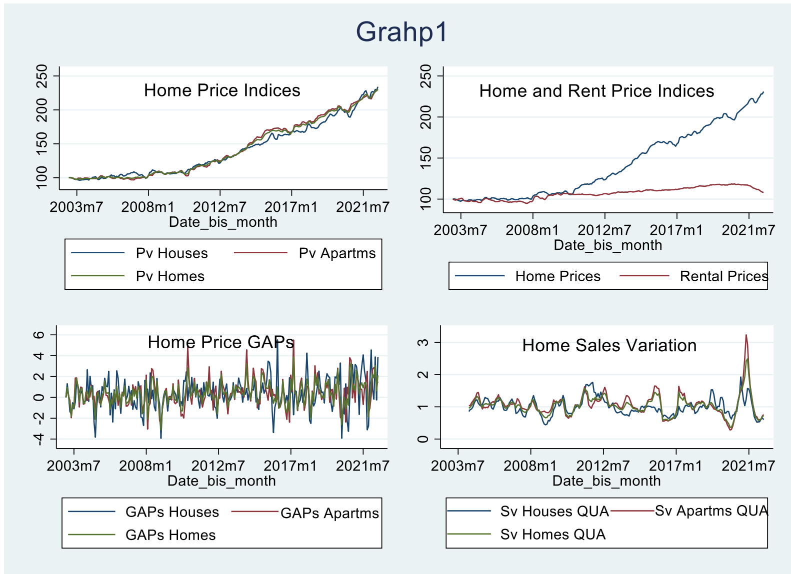
	$\sum GAP$ [1]	$\hat{Y}_{Pv.GAP}$ [2]	*Bubble* [1]x[2]=[3]	$\sum Pv$ [4]	$GAP/\sum Pv$ [3]/[4]=[5]	*GAP Expanded* [6]=1/(1-[5]) -1)
Houses	88.444	0.1599	14.142	134.1	10.50%	11.80%
Apartms	88.606	0.2074	18.377	130.1	14.10%	16.40%
Homes	88.213	0.2484	21.912	131	16.70%	20.10%

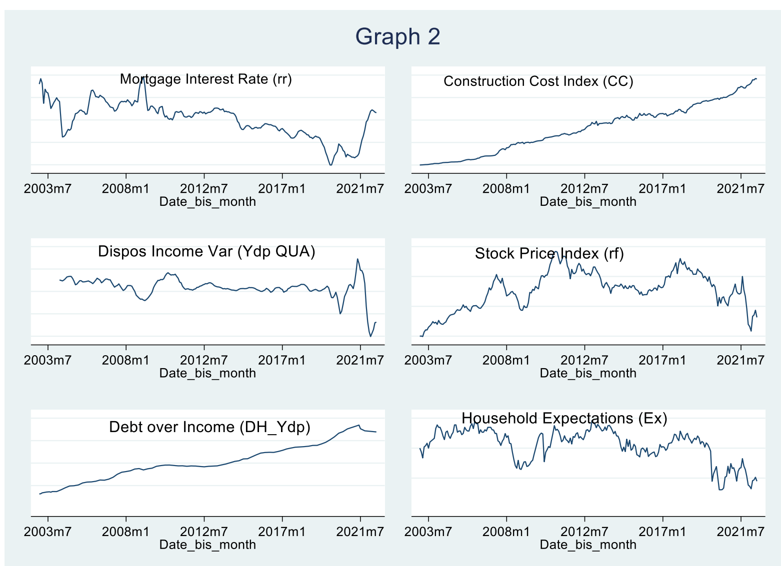
Figures

Figure 1: Zones and Counties of Great Santiago

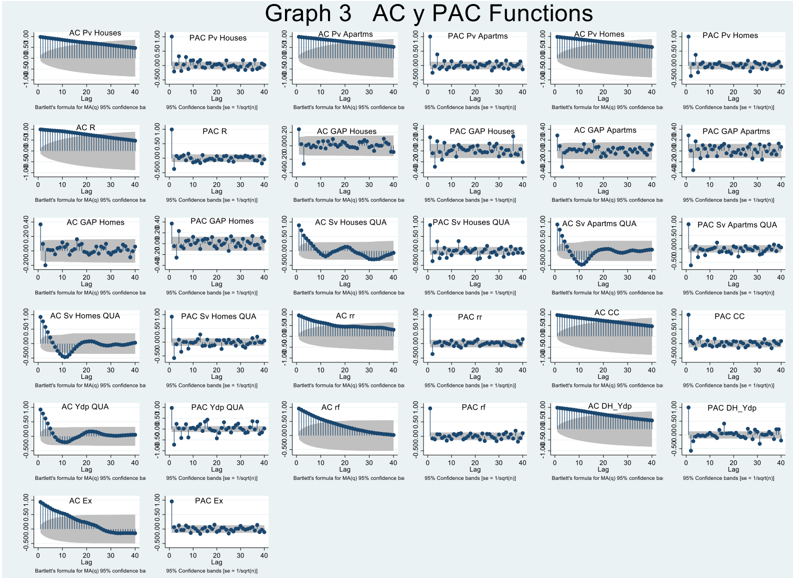
Number	Zone
1	Santiago Downtoun
2	North-West
3	Nor- East
4	South

Graphs

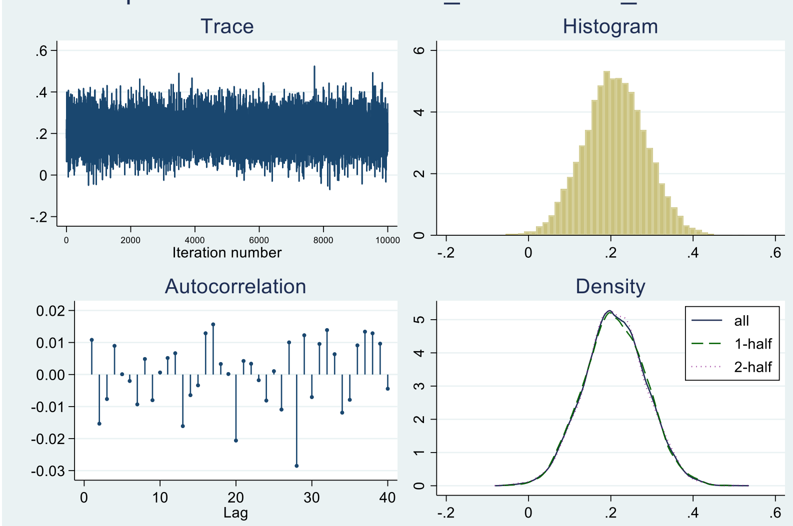

Grahp1
Home Price Indices
Pv Houses
Pv Apartms
Pv Homes
Home and Rent Price Indices
Home Prices
Rental Prices
Home Price GAPs
GAPs Houses
GAPs Apartms
GAPs Homes
Home Sales Variation
Sv Houses QUA
Sv Apartms QUA
Sv Homes QUA
Date_bis_month

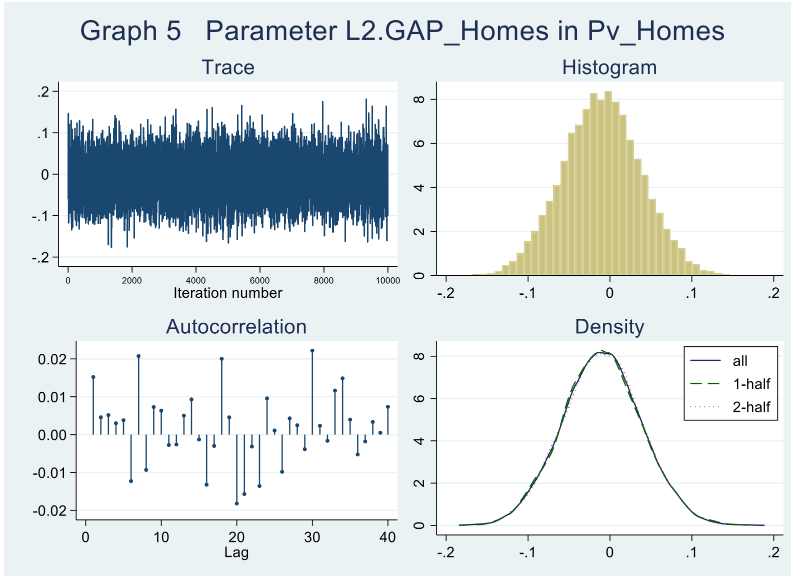

Graph 2
Mortgage Interest Rate (rr)
Construction Cost Index (CC)
Dispos Income Var (Ydp QUA)
Stock Price Index (rf)
Debt over Income (DH_Ydp)
Household Expectations (Ex)
Date_bis_month

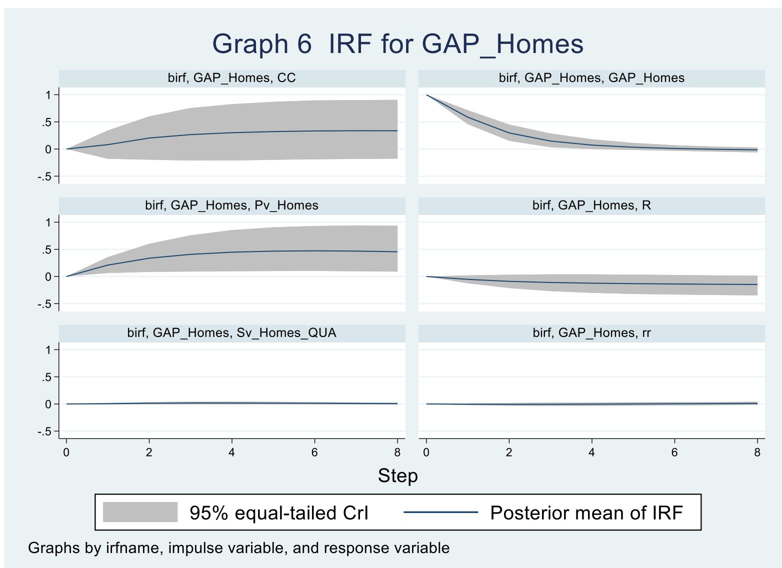
Graph 3 AC y PAC Functions

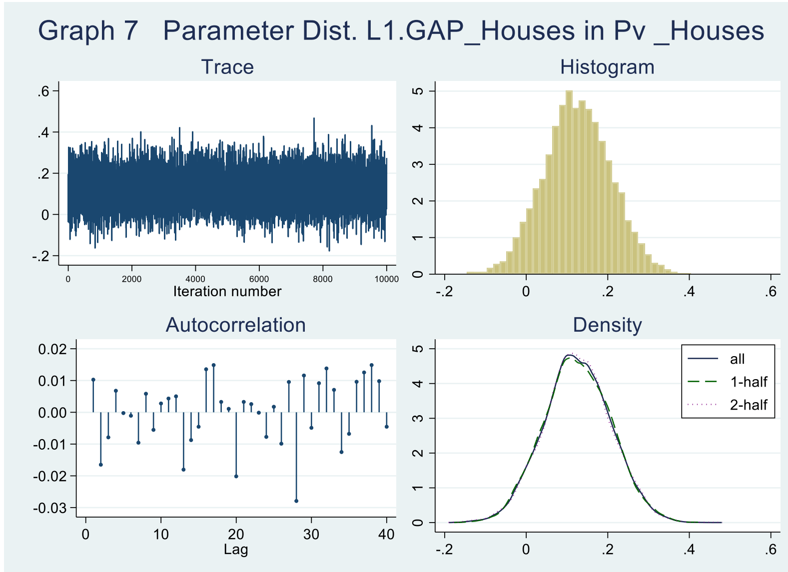


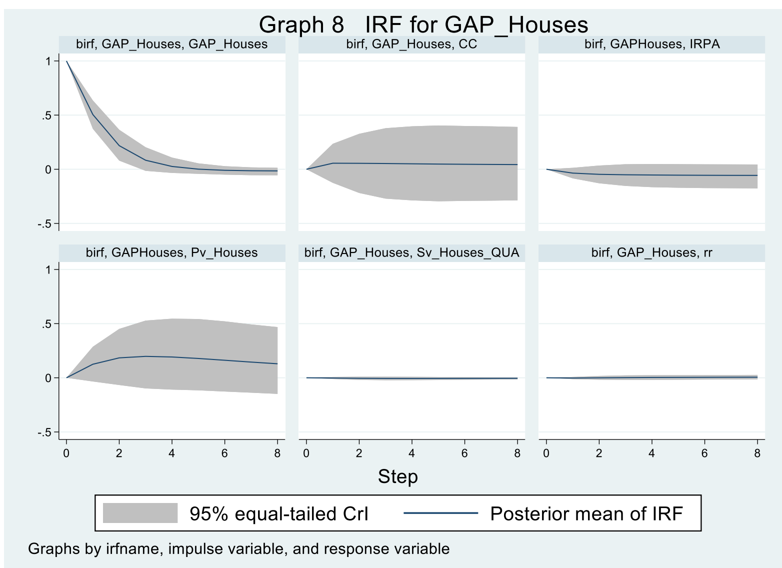
Graph 4 Parameter L1.GAP_Homes in Pv_Homes

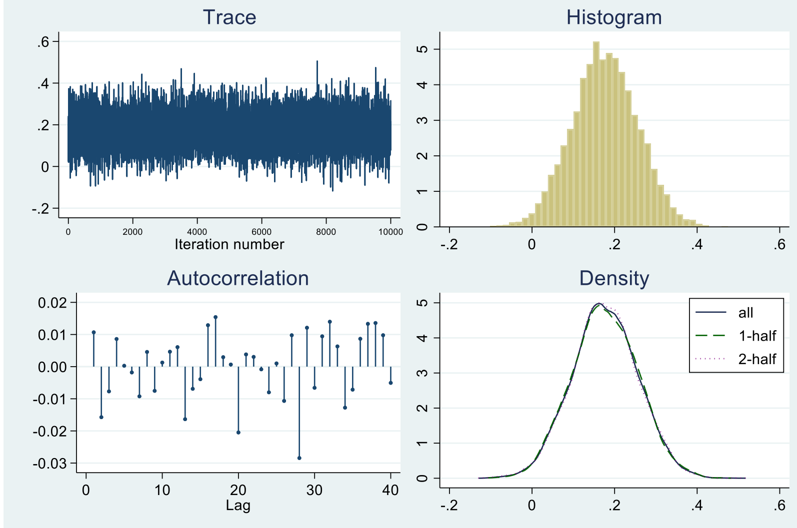



Graph 5 Parameter L2.GAP_Homes in Pv_Homes
Trace
Histogram
Iteration number
Autocorrelation
Density
Lag
all
1-half
2-half

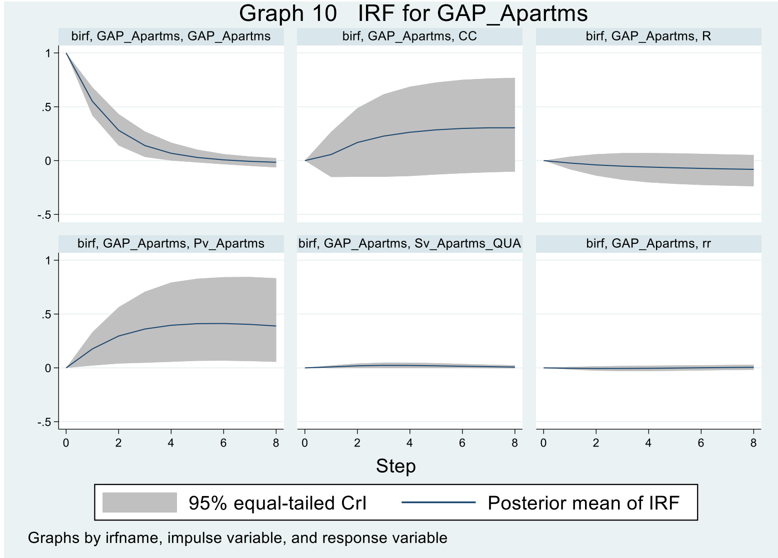

Graph 6 IRF for GAP_Homes
birf, GAP_Homes, CC
birf, GAP_Homes, GAP_Homes
birf, GAP_Homes, Pv_Homes
birf, GAP_Homes, R
birf, GAP_Homes, Sv_Homes_QUA
birf, GAP_Homes, rr
Step
95% equal-tailed CrI
Posterior mean of IRF
Graphs by irfname, impulse variable, and response variable


Graph 7 Parameter Dist. L1.GAP_Houses in Pv _Houses
Trace
Iteration number
Histogram
Autocorrelation
Lag
Density
all
1-half
2-half


Graph 8 IRF for GAP_Houses
birf, GAP_Houses, GAP_Houses
birf, GAP_Houses, CC
birf, GAPHouses, IRPA
birf, GAPHouses, Pv_Houses
birf, GAP_Houses, Sv_Houses_QUA
birf, GAP_Houses, rr
Step
95% equal-tailed CrI
Posterior mean of IRF
Graphs by irfname, impulse variable, and response variable



Graph 9 Parameter Dist. L1.GAP_Apartms in Pv_Apartms



Graph 10 IRF for GAP_Apartms