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Public Goods, Labor Supply and the Source of Economic Distortions¹

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ABSTRACT

The public finance literature claims that any tax instrument other than the lump-sum tax is inherently distortionary because it alters relative prices. This paper suggests otherwise. Conventional time allocation problems typically assume that taxpayers do not consider the individual marginal benefits received in the form of public goods when choosing their labor supply responses to tax policy. Based on the resultant optimality conditions, and assuming that public goods are financed exclusively with (supposedly distortionary) labor income tax, the traditional theory derives the second-best solution to the public goods problem. This paper argues that taxpayers may consider the marginal benefits of taxation in their labor supply responses to taxation, and shows that when they do, the economy can reach an outcome that may be regarded as efficient. Indeed, at that outcome each taxpayer contributes in accordance to the individual benefits received from his/her preferred amount of public goods, and the relative price of leisure remains undistorted.

Keywords: public goods, distortion, Lindahl price, labor tax, lump-sum tax, labor supply

JEL codes: H21, H24, H41, J22

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1. Introduction

In order to determine the optimal amount of (pure) public goods the theory distinguishes the first-best scenario, in which efficient lump-sum taxation is available, from the second-best best scenario, in which the financing of public goods relies on other tax instruments. The first-best solution to the public goods problem is conventionally defined by the vector of lump-sum taxes and transfers that maximizes social welfare. In general, the lump-sum tax is considered as the only efficient or non-distortionary tax instrument, and any other tax is said to prevent the attainment of the first-best solution because it distorts the relative prices in the economy.

This paper suggests that alternative tax instruments other than the lump-sum tax, and in particular the labor income tax, may not be inherently distortionary as it is currently presumed. The argument is twofold. The first part of the argument deals with an implicit assumption made in the traditional taxpayer's time allocation problem, according to which taxpayers disregard the individual marginal benefits of taxation in their labor supply responses to the tax policy. Once this assumption is relaxed for all taxpayers it is possible for the economy to reach a solution that satisfies Samuelson's (1954) condition, traditionally associated with the first-best solution obtained under efficient lump-sum taxes and transfers. This result is not new, as it can readily be obtained in a second-best scenario when utility functions are weakly separable (Christiansen, 1981; Boadway and Keen, 1993) or when the tax elasticity of labor supply is zero, but in this case it holds for any quasi-concave utility function while, in addition, the relative price of leisure remains undistorted (equal to the monetary wage rate) after the tax policy has been implemented.

The second part of the argument deals with what 'should be' the appropriate framework to define economic distortions and efficiency. The first-best solution to the public goods problem does not necessarily provide a convenient framework, since it is obtained by considering both efficiency and equity goals. That solution can be seen as the result of adjusting a "strictly" efficient solution with lump-sum taxes and transfers designed to satisfy the prevailing interpersonal equity conditions. This paper argues that the question about how to define economic efficiency is equivalent to the question about what the strictly efficient solution to the public goods problem should be. One way is to define the efficient solution as the result of maximizing the utility of each taxpayer separately, which avoids interpersonal utility comparisons and excludes any equity consideration. Consequently, the point of maximum individual utility corresponds to the point where the relative price of leisure and the monetary wage rate are equalized by the labor income tax, such that the vector of labor income tax rates is efficient in the traditional sense and equivalent to lump-sum taxes of identical amounts. This approach allows us to define a distributional-independent solution to the public goods problem, as opposed to the distributional-neutral solution proposed by Kaplow (1996, 2008), who makes use of (*ex-post*) income tax adjustments to offset the benefits of tax and expenditure policies.

This proposed solution to the public goods problem provides a useful benchmark to define the concept of economic efficiency in the context of public goods provision and to clearly separate it from the concepts of equity and distortions. In addition, there are other desirable

characteristics of this new approach. Firstly, this solution can be considered as “fair” since it satisfies the benefit principle (along with the Lindahl solution) and the ability to pay principle, at the individual level. The solution is also valid in the presence of impure public goods and other positive restrictions. Finally, if the taxpayer mobility restriction is relaxed, Tiebout’s (1956) sorting process provides an incentive compatibility mechanism by which the proposed efficient solution to the public goods problem holds even in the presence of free riding.

The paper is organized as follows. Section 2 derives the efficiency condition associated with the first-best solution to the public goods problem using the lump-sum tax, and then shows that if taxpayers consider the individual marginal benefits of taxation the same result can be reached with a proportional labor income tax. Section 3 describes the labor supply behavior of a taxpayer that considers those benefits. Section 4 characterizes a strictly individualistic and distributionally-indifferent solution under heterogeneous taxpayers, and describes alternative sources of economic distortions. Finally, the last section concludes.

2. The Solution to the Public Goods Problem in an Economy with Identical Taxpayers

Traditional public finance theory distinguishes between first-best and second-best scenarios based on whether efficient (non-distortionary) lump-sum taxation is available or not to the tax authority. If efficient lump-sum taxation is available the first-best solution to the public goods problem is feasible and satisfies the Samuelson condition, which states that the sum of marginal rates of substitution between public and private goods for all individuals must be equal to the marginal rate of transformation between public and private goods. However, if taxes are collected from any other (presumably distortionary) tax instrument, then the problem automatically turns into finding the second-best solution. This section proposes that the type of tax instrument might not be an adequate criterion to distinguish between the first-best and the second-best scenarios. Specifically, it shows that, in an economy with identical taxpayers, the Samuelson condition reveals the optimal solution to the public goods problem even using a proportional labor income tax, which has been typically considered to be a distortionary (second-best) tax instrument. In order to obtain this result the only theoretical requirement is to allow taxpayers to consider the individual marginal benefits of taxation in their labor supply decisions. This assumption may be debatable, but whether or not this is practical remains an empirical question for future research.

2.1 The traditional first-best and second-best solutions to the public goods problem

Given that taxpayers react to tax policy, the public goods problem can be decomposed in a sequential game of two stages. In the first stage the tax authority sets the level of the tax rate and knows that in the second stage the taxpayers choose their own optimal behavioral responses. In

order to determine the optimal tax rate, therefore, we invoke backward induction and begin by solving the taxpayer's time allocation problem.

The representative taxpayer maximizes his individual utility, ω , described by a quasi-concave function that decreases with the individual supply of labor, l , and increases with the consumption of private goods, x , and public goods, G .² Individual income is assumed to come from the wage rate, w , and from exogenously determined non-labor income, m . The tax rate on labor income is denoted by t , while a tax rate s is applied only on non-labor income and used here to represent the lump-sum tax.³ Assuming no savings, the taxpayer's budget constraint is given by $x = (1 - t)wl + (1 - s)m$, and the taxpayer's time allocation problem, as conventionally presented in the literature, is:

$$\max_l \omega = u\{(1 - t)wl + (1 - s)m, l, G\},$$

where G is assumed to be exogenous and therefore independent from labor supply responses to taxation, tax collections, or the tax rates. The first order condition for this problem is:

$$-\frac{u_l}{u_x} = (1 - t)w, \quad (1)$$

where subscripts are used to represent derivatives with respect to the designated variable. At the optimal time allocation solution the marginal rate of substitution between leisure and private consumption must be equal to the after-tax wage rate.⁴ This condition implies that only t alters the marginal rate of substitution at which the taxpayer finds his optimal time allocation solution. In contrast, the marginal rate of substitution remains unchanged for any $s > 0$. Because of this reason the tax on non-labor income (or the lump-sum tax) is said to impose no substitution effect on the labor decision and is considered as a non-distortionary tax. Indeed, it is easy to show that when the tax authority collects all revenues with the tax on non-labor income then the optimal solution to the problem satisfies Samuelson's condition. In a simple economy with N identical taxpayers and where all revenues are collected with the tax on non-labor income, the welfare maximization problem can be written as:

$$\max_s \Omega = Nu\{wl(s) + (1 - s)m, l(s), sNm\},$$

where Ω represents a social welfare function and $sNm = G$. By omitting a production function we have implicitly assumed that the marginal rate of transformation between public and private goods is constant and equal to one. The first order condition describing the solution to this problem is:

$$wL_s^* - Nm + L_s^* \frac{u_l}{u_x} + Nm \cdot N \frac{u_G}{u_x} = 0,$$

² Individual parameters are represented by lowercases and aggregate parameters by uppercases.

³ According to Atkinson and Stiglitz (1980, p.28) "[l]ump-sum taxes are defined as those that do not depend on any action of the individual; there is no way he can change the tax liability." In an economy with identical taxpayers any lump-sum amount can be represented as a certain proportion of non-labor income.

⁴ The marginal utility of leisure is by definition equal to the negative value of the marginal utility of labor.

where asterisks are used to represent optimal choices and $L_s^* = Nl_s^*$ denotes the aggregate labor supply response to the tax rate. Using the fact that the marginal rate of substitution between leisure and private consumption is constant and equal to w —as in (1) with $t = 0$, then the first order condition for this welfare maximization problem can be reduced to:

$$N \frac{u_G}{u_x} = 1, \quad (2)$$

which is Samuelson's condition for an economy where the marginal rate of transformation between public and private goods is one.

Under the traditional framework the use of a labor income tax leads to a deviation from Samuelson's condition. Assuming that the tax authority collects all its revenues from the tax on labor income the welfare maximizing problem becomes:

$$\max_t \Omega = Nu\{(1-t)wl(t) + m, l(t), tNwl(t)\},$$

where $tNwl(t) = G(t)$. The first order condition for this problem is:

$$-wL + (1-t)wL_t^* + L_t^* \frac{u_l}{u_x} + \{wL + twL_t^*\} N \frac{u_G}{u_x} = 0; \quad (3)$$

and using (1) this condition can be reduced to:

$$N \frac{u_G}{u_x} = \frac{wL}{wL + twL_t^*}.$$

This equation corresponds to the traditional second-best solution to the public goods problem as described in the literature on the marginal cost of public funds.⁵ The value of the marginal cost of public funds is given by the expression in the right hand side, which depends upon the labor supply response to taxation. If the labor supply is reduced by a marginal increase of the tax rate ($L_t^* < 0$), then the sum of the marginal rates of substitution between public and private goods is greater than one. The literature also considers the possibility of observing a positive labor response to taxation, such that the marginal cost of public funds can also be lower than one (Stiglitz and Dasgupta, 1971; Atkinson and Stern, 1974). In particular, when the (uncompensated) labor supply is inelastic with respect to the tax rate the expression for the second-best solution equals the Samuelson condition in (2). This special case is not interpreted in the literature as an efficient outcome; instead, it has led to a discussion about the relationship between the marginal cost of funds and the concept of excess burden of taxation (Triest, 1990).

2.2 Obtaining the Samuelson condition with a proportional labor income tax

The labor supply response to taxation is traditionally modeled as a time allocation problem where the taxpayer chooses the optimal combination of private consumption and labor for any given after-tax wage rate. Public goods, if considered in the taxpayer's preferences, are assumed to be

⁵ See, for instance, Hakonsen (1998), and Auerbach and Hines (2002) and Dahlby (2008).

constant or exogenously determined. This is equivalent to assume that even though the taxpayer might react to a change in the provision of public goods, the taxpayer does not consider the effect of a marginal change in the tax rate on revenue collections and public goods provision.

A marginal tax rate increase, however, is usually imposed over many taxpayers and thus tax collections can significantly increase with the tax reform. The tax increase leads not only to a reduction in a taxpayer's disposable income but also to an increase in public goods provision, which in turn improves his/her standard of living and thus might lead to a reevaluation of the labor supply decision. It should not matter whether the taxpayer is aware of the connection between the tax increase and the greater provision of public goods, or whether the latter is realized soon after the former. The taxpayer would consider the change in public goods provision simply because he/she values them, and the utility maximizing central planner would consider the full taxpayer response because it is relevant for tax and expenditure policies.

In order to incorporate the effect of an increase in marginal public goods in the taxpayer's labor response to taxation we need only to rewrite the time allocation problem as:

$$\max_l \omega = u\{(1-t)wl + (1-s)m, l, tNwl + sNm\},$$

where $tNwl + sNm = G$, such that public goods are allowed to vary with the labor supply responses of the N taxpayers to the tax rates. Provided that the N identical taxpayers modify their labor supply decisions simultaneously, the first order condition for the representative taxpayer becomes:

$$-\frac{u_l}{u_x} = (1-t)w + tNw \frac{u_G}{u_x}, \quad (4)$$

where the right-hand-side is equal to the "full" real wage rate. The marginal rate of substitution between leisure and private goods is no longer equal to the after-tax monetary wage rate—as in (1), because it considers the individual welfare gains due to greater public goods provision. This can be regarded as the "correct" measure of the real wage rate because it encompasses all individual marginal gains associated with the last unit of leisure enjoyed by the taxpayer.

Just like the traditional formulation of the time allocation model, the tax rate on non-labor income (s) does not appear to affect the (full) optimal real wage rate. However, now the real wage rate is a function of the quantity of public goods available to the taxpayer.⁶ This functional dependency has important consequences for the optimality solutions of both the taxpayer's time allocation problem and the public goods problem. The assumption that the taxpayer considers the marginal benefits of taxation may change the predicted labor response to the tax rate. In turn, this modified labor response to the tax rate alters the optimal solution to the public goods problem: Substituting (4) into (3) and rearranging terms, the new first order condition to the problem for an

⁶ Relatively scarce public goods (and abundant private goods) will likely be associated with a high marginal utility u_G (and low u_x), such that the marginal rate of substitution between public and private goods might be expected to be large. If this marginal rate of substitution is large enough, then marginal full real income can exceed the wage rate, w , a result that is not possible under conventional labor supply theory.

economy with identical taxpayers is the traditional Samuelson's condition of (2). A similar result has already been found by Christiansen (1981) and Boadway and Keen (1993), but they require public and private goods to be weakly separable from leisure in the utility functions. Here, however, no assumption about the functional form of the utility function is necessary. This result is at odds with the current public finance theory because it suggests that the proportional labor income tax might not be inherently distortionary as it is traditionally presumed. Note that together (2) and (4) imply that the full real wage rate at the optimal solution is w –which is the same value obtained under lump-sum taxation.

3. Taxpayer's Labor Supply Response to the Proportional Labor Income Tax

The traditional time allocation model assumes that the taxpayer disregards the effect of taxation on the provision of public goods. A marginal tax increase, however, translates into more tax collections and public goods provision, which in turn is equivalent to an increase in taxpayer's standard of living and thus may influence his/her labor supply decision. This section analyzes the response to the labor income tax when the taxpayer perceives and considers the change in public goods provision that follows a tax rate increase. Under this assumption the maximization problem for taxpayer j is:

$$\max_j \omega^j = u^j \{ (1-t)w^j l^j + m^j, l^j, t \sum_{i=1}^N w^i l^i \},$$

where wage rate, non-labor income and labor supply have been allowed to differ for any taxpayer i . Considering that the labor decision of all taxpayers is made simultaneously, the first order condition for taxpayer j can be expressed as:

$$-u_l^j = (1-t)w^j u_x^j + t [\sum_{i=1}^N w^i] u_G^j, \quad (5)$$

where $\sum_{i=1}^N w^i$ is the marginal change in the tax base due to the marginal change of aggregate labor supply, and whose value would become measurable only once all taxpayers responses to the tax policy are observed. From (5), the last unit of leisure allows each taxpayer to enjoy his after-tax monetary income plus the monetary value he assigns to the marginal benefits received in the form of public goods provision. Dividing by u_x^j and arranging terms we can solve for the taxpayer j 's marginal rate of substitution between leisure and private consumption:

$$-\frac{u_l^j}{u_x^j} = w^j + t \left\{ [\sum_{i=1}^N w^i] \frac{u_G^j}{u_x^j} - w^j \right\}.$$

When t is zero the marginal real income is simply w^j , but as t increases it first multiplies a (presumably) high marginal rate of substitution between public and private consumption, such that the marginal change in full real income due to the tax rate increase can be expected to initially exceed the nominal wage rate, w^j . In that case taxation would be associated in the margin with net real income gains. As the tax rate increases public goods become relatively more

abundant and private goods more scarce; as a consequence, the individual marginal rate of substitution between public and private goods decreases, as does the full real wage rate, which eventually falls below w^j . In general, if the full real wage rate is smaller than the nominal wage rate, then a marginal tax increase leads to a net reduction of full real income.

The first order condition of (5) implicitly defines taxpayer j 's labor behavior. Using the implicit function theorem we can find that the labor response to taxation is:

$$l_t^{j*} = \frac{u_x^j}{-\omega_{ll}^j} \left\{ \left[\sum_{i=1}^N w^i \right] \frac{u_G^j}{u_x^j} - w^j \right\} \quad (6.a)$$

$$- \frac{w^j l^j}{-\omega_{ll}^j} \{ (1-t) w^j u_{xx}^j + u_{lx}^j + t \left[\sum_{i=1}^N w^i \right] u_{Gx}^j \} \quad (6.b)$$

$$+ \frac{\sum_{i=1}^N w^i l^i}{-\omega_{ll}^j} \{ (1-t) w^j u_{xG}^j + u_{lG}^j + t \left[\sum_{i=1}^N w^i \right] u_{GG}^j \} , \quad (6.c)$$

where $-\omega_{ll}^j > 0$ by the second order condition of the taxpayer's maximization problem. When the public goods provision is allowed to vary with the tax rate the labor response to taxation comprises three effects, separately described by (6.a), (6.b), and (6.c).⁷ (6.a) represents a "net substitution effect," whose sign would be positive or negative depending on whether the marginal tax increase is associated with a net real income gain or a net real income loss, respectively. If the full real wage rate is greater (smaller) than the nominal wage rate, then the taxpayer would increase (reduce) labor supply.

Equations (6.b) and (6.c) represent a "private income effect," and a "public goods income effect," respectively. The private income effect describes how labor supply reacts to the reduction of (an equivalent amount of) non-labor income available to purchase private goods, which is similar to the income effect described by the traditional literature.⁸ The public goods income effect describes how the labor supply reacts to the increase of (an equivalent amount of) non-labor income available in the form of public goods benefits. The private and public goods income effects can be considered as two components of a sole net real income effect. Whenever the equivalent income gain from additional public goods is greater (smaller) than the loss in monetary disposable income, full real income will be increased (decreased) with the tax rate. Full real

⁷ In the traditional time allocation model, the labor supply response to taxation consists of two effects. The substitution effect is always negative since the tax reduces net marginal income. On the other hand, the income effect can be positive or negative depending on whether leisure is normal or inferior, respectively. The sign of the labor supply response to taxation is ambiguous, although it is common to assume that leisure is a normal good but the positive income effect of the tax is not big enough to counterbalance its negative substitution effect. Surveys on labor supply theories and empirical results can be found, for instance, in Blundell and MaCurdy (1999), and Meghir and Phillips (2010).

⁸ Note that the term $t \left(\sum_{i=1}^N w^i \right) u_{Gx}^j$ in (6.b) is not considered in the income effect described by the traditional labor supply theory. This term would be positive if an additional unit of the public goods increases the benefits received by the last unit of private good consumed. In such a case, the private income effect would be smaller than the one estimated by the traditional theory.

income is maximized at the point where the full real wage rate is equal to the nominal wage rate, where the net substitution effect is zero. Moreover, full real income can be assumed as a monotonic transformation of individual utility, such that the same point would also correspond to the maximum individual utility.

The net real income effect might be expected to have the opposite sign of the net substitution effect: Larger (smaller) full real income makes the next unit of leisure more (less) enjoyable and this would lead to greater demand for leisure.⁹ However, in order for a unique and well defined individual utility maximum to exist the net substitution effect must dominate over the real income effect. If full real income is defined as the product of full real wage rate and the labor supplied, simultaneous maximization of full real income and utility requires the labor supply to be maximized at the same point.

The shape of the characteristic indifference curves and the resultant labor supply function under this framework are depicted in Figure 1, where the horizontal axis represents leisure, $\ell(t)$, and the vertical axis private consumption, $x(t)$. Total time available is fixed at T and the amount of leisure implicitly defines the labor supply as the horizontal distance at the left of T . The fact that the maximum value of real income is associated with a positive tax rate implies that the preferences for private goods and leisure are not insatiable, which the conventional time allocation problem assumes. When the labor income tax rate is zero the taxpayer chooses point e_0 , where the indifference curve I is tangent to the taxpayers' budget constraint and the optimal combination of private consumption and leisure is described by $[x^*(0), \ell^*(0)]$.

Figure 1: Labor supply response to the proportional labor income tax

⁹ This reasoning is consistent with the usual assumption about the normality of leisure, where the taxpayer chooses to increase leisure as a response to a marginal increase of (lump-sum) income.

A marginal increase of the tax rate initially increases labor supply because the marginal welfare benefits exceed the marginal welfare costs of taxation. Labor supply continues to increase until the point where the marginal benefits and costs of taxation are identical, which is found when the tax rate is t_1 . Further increases of the tax rate reduce the level of utility, and as the level of utility decreases the taxpayer also reduces the amount of labor supplied, since the additional effort becomes less rewarding.

4. Efficient Provision of Public Goods with Heterogeneous Taxpayers

This section utilizes the labor supply model developed above to derive a solution to the public goods problem that is independent from any equity considerations and thus requires no interpersonal comparisons of utility. Considering taxpayers with different preferences, wage rates and non-labor income, it is shown that if each taxpayer considers the individual marginal benefits of taxation on labor behavior the efficiency condition traditionally associated with a first-best solution can also apply to a tax on labor income. Based on this result we can argue that the tax on labor income is an efficient tax.

Each taxpayer i is assumed to maximize an individual utility function u^i , subject to the budget constraint $x^i = (1 - t^i)w^i l^i(t^i) + m^i$. Taxpayers are organized in a sequence of (ascending) relative preferences for public goods, which are here defined in accordance to the tax rate at which a taxpayer reaches the maximum attainable level of utility given other taxpayers' contributions to the public goods. Taxpayer $i = 1$ maximizes utility with the lowest tax rate, while taxpayer $i = N$ maximizes utility with the greatest tax rate. An omniscient central planner can maximize each taxpayer i 's utility by choosing the tax rate t^{i**} that maximizes individual utility.

Initially, a common tax rate can be applied to all taxpayers and be set at the optimal level for taxpayer 1. At this tax rate, denoted as t^{1**} , only taxpayer 1 is fully satisfied with the amount of public goods provided, and all other taxpayers would prefer a tax increase. The optimal tax policy in this situation is to impose t^{1**} for only taxpayer 1 and to increase the tax rate for the remaining $N - 1$ taxpayers. Note that taxpayer 1's labor supply remains at its maximum and the labor supply of all other taxpayers is necessarily increasing with the tax rate. The next step is to increase the tax rate up to t^{2**} , the level at which taxpayer 2 maximizes his/her level of utility. Now the optimal tax policy is to impose t^{2**} for only taxpayer 2 and to increase the tax rate for the remaining $N - 2$ taxpayers. The same procedure can be repeated N times such that all taxpayers reach their maximum attainable level of utility and supply the maximum amount of labor under the labor income tax.

Formally, the problem of finding the optimal tax rate for any taxpayer n can be written as:

$$\max_{t^n} \omega^n = u^n\{(1 - t^n)w^n l^n(t^n) + m^n, l^n(t^n), \sum_{i=n}^N t^i w^i l^i(t^i)\} ,$$

where marginal public goods provision is assumed to be financed only by taxpayer n and the $N - n$ taxpayers with greater preferences for public goods. The first order condition for this maximization problem is:

$$-w^n l^n + (1 - t)w^n l_t^{n*} + l_t^{n*} \frac{u_l^n}{u_x^n} + [\sum_{i=n}^N (w^i l^i + t w^i l_t^{i*})] \frac{u_G^n}{u_x^n} = 0. \quad (7)$$

Using the taxpayer's first order condition in (5) and rearranging terms we obtain:¹⁰

$$\frac{u_G^n}{u_x^n} = \frac{w^n l^n}{\sum_{i=n}^N w^i l^i}; \quad (8)$$

condition stating that at the optimal solution for the taxpayer n , the marginal rate of substitution between public and private goods must be equal to the share of the labor income tax base. To simplify, the optimal contribution of any taxpayer n to the financing of public goods is computed as the proportion of his/her share on the marginal utility of the $N - n + 1$ contributing taxpayers, which essentially describes Lindahl's (1958) pricing mechanism. Moreover, for any given t^{n**} , adding up the marginal rate of substitution between public and private goods of the $N - n + 1$ contributing taxpayers we obtain that:

$$\sum_{n=1}^N \frac{u_G^n}{u_x^n} = 1,$$

which is Samuelson's condition to the public goods problem for an economy of $N - n + 1$ heterogeneous taxpayers.

The solution described in (8) for taxpayer n , therefore, satisfies both Lindahl's solution and Samuelson's condition to the tax and expenditure problems, but it consists of as many as N such solutions. A taxpayer contributes to the marginal increases of public goods provision as long as he/she positively values those increases, and stops contributing when public goods provision reaches the level that is optimal. As a result, no taxpayer would have to sacrifice private consumption in order to finance unwanted quantities of public goods, and no taxpayer would be willing but unable to contribute more to the provision of public goods.

Different from the traditional Lindahl solution, in this price mechanism each taxpayer's optimal share is computed at the most preferred amount of public goods, and "reveled" by their labor supply behavior. Indeed, provided that the net substitution effect of the labor responses to taxation dominates over the real income effect, t^{n**} must necessarily be set at the point where the

¹⁰ The first order condition for the taxpayers (5) can be rewritten to consider the discrete changes of labor supply dl for those taxpayers between n and N that would actually be contributing to the public goods:

$$(1 - t)w^n dl^n u_x^n + dl^n u_l^n + t[\sum_{i=n}^N w^i dl^i] u_G^n = 0.$$

Dividing this condition by dt , which is common for all taxpayers between n and N , we obtain:

$$(1 - t)w^n l_t^{n*} u_x^n + l_t^{n*} u_l^n + t[\sum_{i=n}^N w^i l_t^{i*}] u_G^n = 0,$$

which is the expression substituted into (7) in order to obtain (8).

labor supply of taxpayer n is inelastic with respect to the labor income tax rate and reaches its maximum level in the presence of this tax instrument.¹¹

This solution contains characteristics that can be considered as desirable for the purpose of defining economic efficiency. First, it satisfies, in an individual basis, both the benefit principle and the ability-to-pay principle. The benefit principle is satisfied along with the Lindahl's price mechanism, because each taxpayer contributes in accordance with his individual benefits. For instance, taxpayers that do not positively value marginal increases of public goods provision are exempted from the payment of the tax. On the other hand, the ability-to-pay principle is satisfied because the taxpayer's tax burden is paid by means of voluntary labor supply increases and only as long full real income increases with the tax rate.¹²

Second, since the utility of each taxpayer is maximized separately, the solution requires no interpersonal comparisons of utility or (non-economic) equity considerations. Income tax adjustments as proposed by Kaplow (1996, 2008) are not necessary to obtain distribution-neutral solutions to the public goods problem. This is due to the fact that the efficient solution simply avoids the problems related with the aggregation of preferences. Finally, for the same reason the Arrow's (1950) theorem asserting the impossibility of integrating the heterogeneous preferences into a unique well defined social welfare function is not applicable to the efficient solution. As a consequence, the scope of that theorem can safely be restricted to the realm of distributive policies, while allocation efficiency remains (at least in principle) as a "solvable" economic matter.

4.1 Possible deviations from the efficient solution

Absent the ideal conditions under which the efficient solution to the public goods problem is feasible, the task becomes one of finding a second-best solution that accounts for and offers a correction to existing distortions. The previous discussion suggests that by assuming taxpayers disregard the individual marginal benefits of taxation in their labor supply decisions the conventional theory implicitly introduces a distortion in the model that prevents the attainment of the efficient solution. The following discussion introduces additional sources of distortions or deviations from the efficient solution that may be present even when taxpayers behave efficiently.

One source is given by those cases in which public expenditures are not fully devoted to the satisfaction of all taxpayers' preferences. We can use the parameter $\rho \in [0,1]$ to represent the share of public expenditures that is actually used to finance public goods, such that $1 - \rho$ would correspond to the share of public funds diverted from that use (e.g. corruption, production inefficiencies). Similarly, we can define $\gamma^j \in [0,1]$ as the share of public goods that effectively

¹¹ Knowing that the government sets the individual tax rate after observing labor supply behavior a taxpayer might supply less labor in the expectation of enjoying the same public goods while consuming more private goods. This incentive compatibility problem is discussed at the end of the section.

¹² The ability to pay principle is usually defined on an interpersonal basis, which leads to a number of difficulties that are avoided in the proposed framework. For a discussion, see Atkinson and Stiglitz (1980, 260-263).

satisfies taxpayer j 's preferences. The N -vector of γ^j can be considered as a description of the “purity” of public goods. Considering these two parameters, the problem of choosing the tax rate that maximizes taxpayer j 's utility is given by:

$$\max_t \omega^j = u^j\{(1-t)w^j l^j(t) + m^j, l^j(t), \rho \gamma^j t \sum_{i=1}^N w^i l^i(t)\},$$

and the first order condition can be reduced to:

$$\gamma^j \frac{u_G^j}{u_x^j} = \frac{w^j l^j}{\rho \sum_{i=1}^N w^i l^i}. \quad (9)$$

A reduction in the value of γ^j implies that taxpayer j receives less satisfaction from a given amount of public goods. Given that this is a subjective individual effect then it is convenient to place γ^j in the left hand side of the equation, which represents the individual marginal benefits of taxation. In contrast, a smaller ρ means that any given amount of public goods becomes more expensive for all taxpayers, and thus it is convenient to place this parameter in the right hand side of the equation, representing the marginal contribution to the public good. In general, reducing either γ^j or ρ would also reduce the optimal provision of public goods for taxpayer j .

Another source of deviations from the efficient solution is given by a constraint in the quantity of tax rates (or tax brackets) that the central planner is able to implement. If the tax authority is unable to set the tax rate that maximizes utility for each individual then the choice of the tax policy will require the application of interpersonal comparisons of utility and implicit income redistributions due to the need of determining winners and losers. In this case the labor income tax would necessarily distort the relative price of leisure for all taxpayers facing a non-optimal (individual) tax rate and the overall output of the economy would be smaller than that of the efficient solution. It follows that if taxpayers are adequately ordered by their relative preferences for public goods, the introduction of a new tax bracket necessarily provides gains in individual utility and net (Pareto-superior) welfare gains for society. In practice, the optimal public policy would be found at the point where the marginal gains from introducing a new tax category are equal to the marginal costs incurred plus the loss assigned to the distributional effects of this measure.

4.2 The incentive compatibility problem

Just like the traditional Lindahl price mechanism, the proposed efficient solution may be subject to an incentive compatibility problem. If taxpayers know that their individual tax rates are set at the point of inelastic labor supply responses to the labor income tax rate they might alter their behavior in order to take advantage of the positive externalities associated with the public goods. By allowing for the presence of impure public goods, however, we are able to describe a mechanism by which the proposed solution is not only valid in the normative realm of economic theory but also provides a positive solution to the public goods problem.

Note that the gains from free riding are lower for those taxpayers preferring more public goods, who would therefore have fewer incentives to reduce their labor supply. If mobility is possible at a reasonable cost, they would have incentives to group themselves in geographical clusters in order to take advantage of economies of scale and maximize the benefits they receive from the (impure) public goods.¹³ Different clusters may offer different expenditure (and even tax) policies, triggering a Tiebout (1956) sorting process as taxpayers move toward areas where their preferences for public goods are better matched. In such a context, the prices faced by the taxpayers (either as explicit tax rates or implicitly as greater costs of living determined by market mechanisms) are unavoidable and thus no further attempts to reduce the tax burden should be observed. In addition, as long as the taxpayers gain from moving into a new area they would voluntarily choose to work more and pay more taxes.

This sorting process would not end the free riding problem, but it would likely limit its scope. In the end, the efficient solution to the public goods problem would still be described by the optimal condition (9), where γ^j could again be interpreted as a measure of the extent to which public goods satisfy taxpayer j 's preferences, but now adjusted by his/her actual free riding behavior.

5. Conclusions

The traditional first-best solution to the public goods problem is said to hold if and only if efficient lump-sum taxation is available. This paper suggests otherwise, and explains that the traditional conclusion depends upon the assumption that taxpayers do not consider the individual marginal benefits of taxation in their labor supply decisions. Once that assumption is relaxed, and if distributional considerations are excluded, the labor income tax can be used to obtain a solution to the public goods problem that can be regarded as efficient. At this efficient solution it is not relevant whether the optimal tax burden is collected as a lump-sum amount or through a tax on labor income, and the latter does not affect the relative price of leisure.

The proposed efficient solution corresponds to the point where utility is maximized for each individual. The main difference with respect to the traditional approach is that here we identify the precise share of public goods provision that maximizes each taxpayer's utility, and use it to determine the individual tax burden. This solution possesses several desirable properties that make it attractive as a benchmark to defining economic efficiency and distortions. It satisfies both the benefit principle and the ability-to-pay principle of taxation, which implies that taxpayers pay a "fair" price for their consumption of public goods. By construction, the efficient solution requires no interpersonal utility comparisons and thus faces no aggregation problems, and is not subject to an efficiency-equity trade off because it does not require distributional considerations.

¹³ These taxpayers may plausibly form economic clubs or move into geographical areas where zoning arrangements are being implemented, but explicitly excluding other taxpayers is unnecessary under the assumption that the other taxpayers are simply not interested in additional public goods.

Furthermore, given that taxpayers' willingness to pay for public goods is revealed from their labor supply behavior, it is not necessary to trust them to truthfully disclose that information. Some taxpayers may attempt to free ride by reducing their labor supply, but the presence of impure public goods and market mechanisms ensure that the gains of free riding are limited. In any case, the proposed efficient solution to the public goods problem holds regardless the extent of the free riding problem.

An important consequence of using the proposed efficient solution as a benchmark for defining the concept of economic distortion is that the labor income tax appears to be an efficient tax instrument, provided that taxpayers consider the marginal benefit of taxation in their labor supply decisions. This is because under the labor income tax the relative price of leisure remains unchanged at the efficient solution to the public goods problem.

This paper finds that tax instruments different from the lump-sum tax might not be inherently distortionary as traditional theory presumes. The consequences of this conclusion go far beyond the scope of the paper, and are ultimately related with the very concept of economic efficiency. In particular, it casts doubts about the appropriateness of using lump-sum taxation as a standard of reference to identify and measure the distortionary effects of other tax instruments. Moreover, given that the tax on labor income allows for the relative price of leisure to change with the tax rate, then assuming that the "right" price of leisure remains unresponsive to the tax rate might not be indisputably correct. After all, if public goods are valuable for society then the "right" relative prices in the economy should not be those prevailing in the absence of public goods, but instead those obtained after taxes have been collected.

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